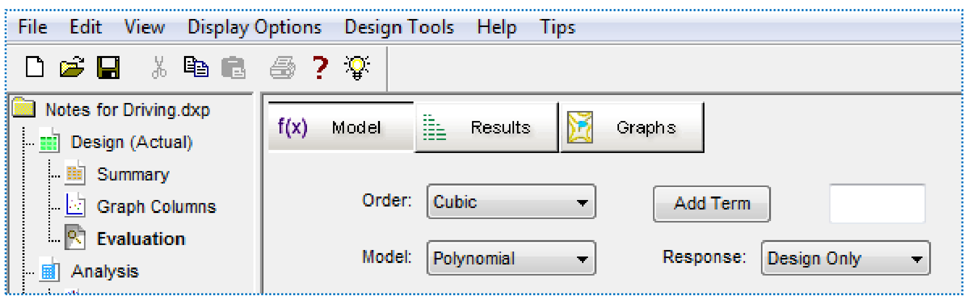
One-Factor RSM Tutorial
(Part 2 – Advanced topics)

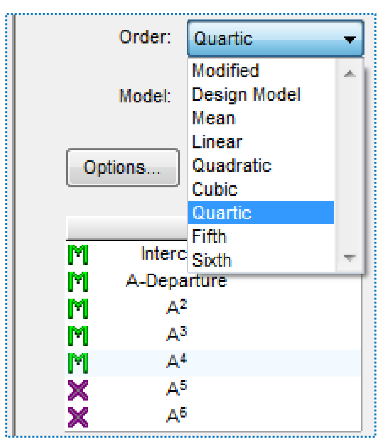
Adding Higher-Order Model Terms

If you still have the driving data active in Design-Expert® software from Part 1 of this tutorial, continue on. If you exited the program, re-start it using our new opening screen (click **Open Design**) or use **File**, **Open Design** to open data file **Drive time.dxp**. Otherwise, go back and set it up as instructed in One-Factor RSM Tutorial (Part 1 – The Basics). The wavy curve you see on the response surface plot for drive time is characteristic of a third-order (cubic) polynomial model. Could an even higher-order model be applied to the data from this case? If so, would it improve the fit? Under the **Design** branch click the **Evaluation** node.



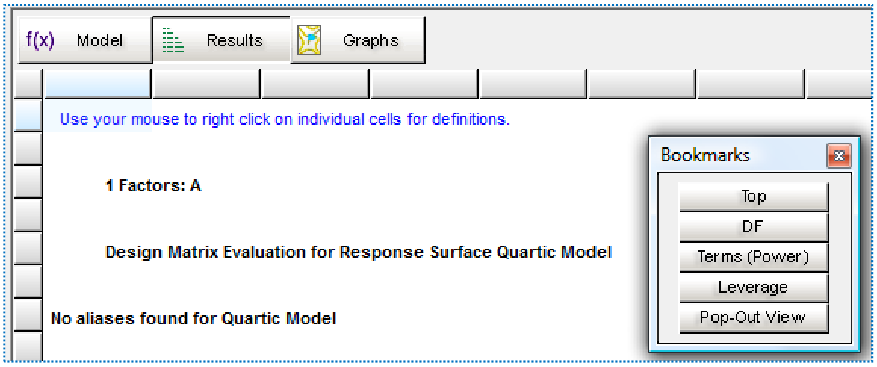
Design evaluation

Change the **Order** to **Quartic** or double-click the term A^4 to put it in the model (“M”).



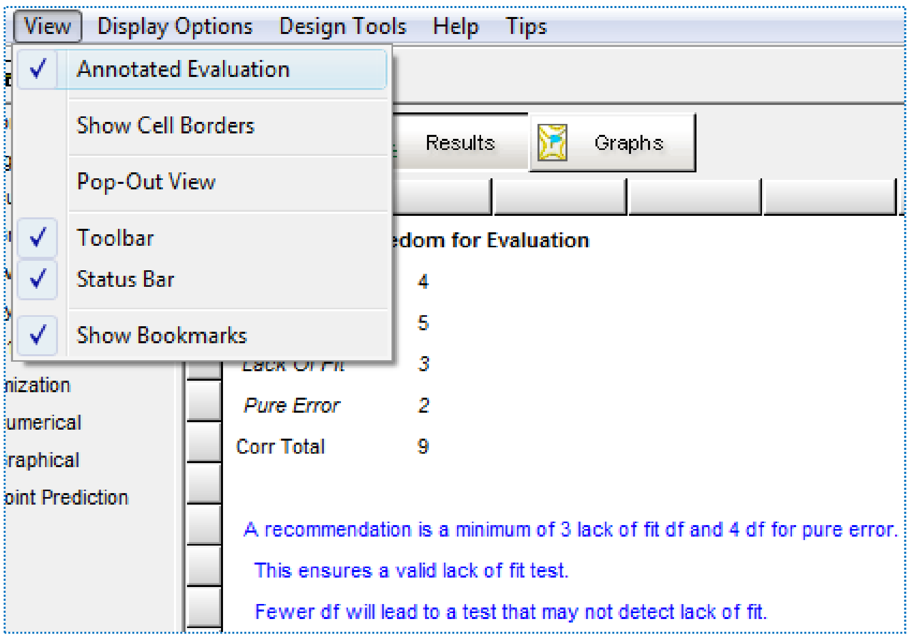
Model changed to quartic (4th order)

Click ***Results*** to see the evaluation of this higher-order model.



Evaluation finds no aliases for quartic model

No aliases are found, but other aspects of the evaluation fall short of the ideal. Scroll down the output (or use the Bookmarks) and pay close attention to the annotations. If you do not see these on-screen, go to View and select Annotated Evaluation.



Annotated view – degrees of freedom detailed

Scroll down further to see statistics on power. Reading the annotations below, you will realize that going to the quartic model may not be such a good idea.

				Power at 5 % alpha level to detect signal		
Term	StdErr**	VIF	Ri-Squared	0.5 Std. Dev.	1 Std. Dev.	2 Std. Dev.
A	1.56	15.78	0.9366	5.2 %	5.8 %	8.3 %
A^2	2.14	14.15	0.9293	5.4 %	6.7 %	12.0 %
A^3	1.78	29.86	0.9665	5.2 %	5.6 %	7.5 %
A^4	1.69	28.30	0.9647	5.7 %	7.8 %	16.3 %

**Basis Std. Dev. = 1.0

Standard errors should be similar within type of coefficient. Smaller is better.

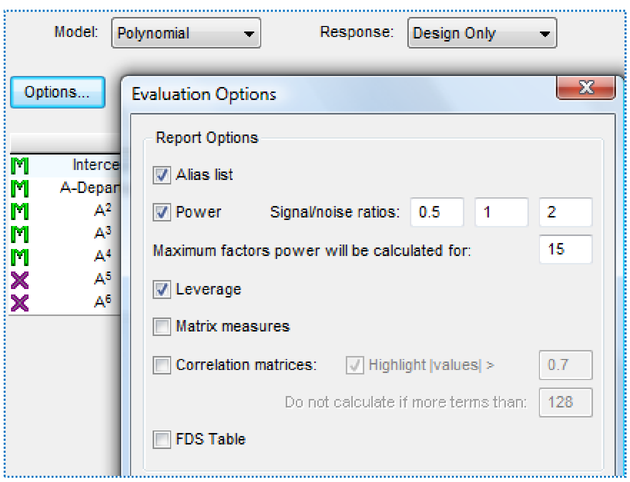
Ideal VIF is 1.0. VIF's above 10 are cause for alarm,
indicating coefficients are poorly estimated due to multicollinearity.

Ideal Ri-squared is 0.0. High Ri-squared means terms are correlated with each other,
possibly leading to poor models.

Power statistics

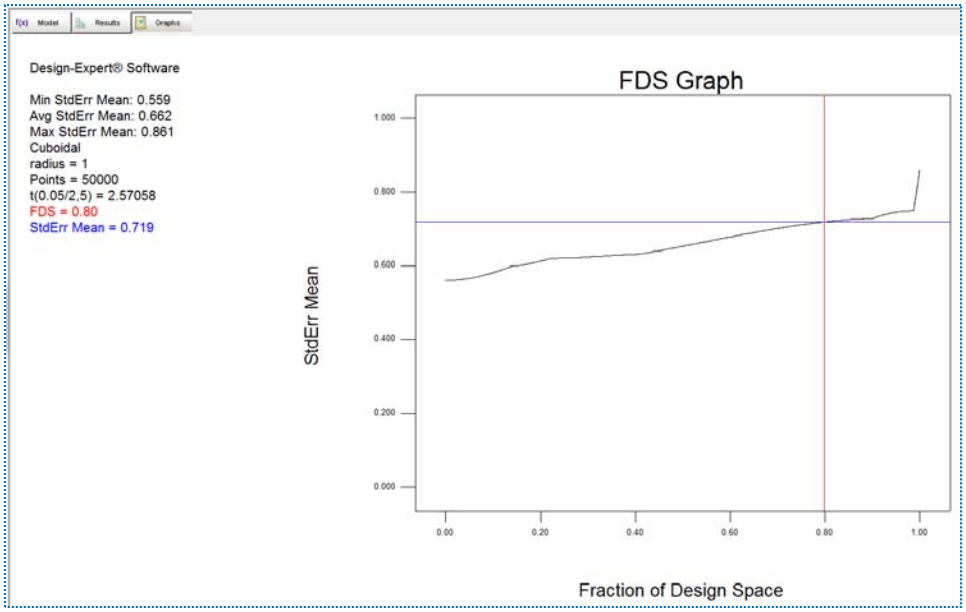
Scroll to the bottom of this report and view the leverages. Note the design point with the unusually high leverage of 0.9743. This is the late departure time near 50 minutes that occurred due to Mark oversleeping. This is what DOE experts refer to as a 'botched' factor setting. It should not be surprising to come out poorly for leverage.

Many more evaluation statistics can be generated from Design-Expert if you like – the ones shown by default are the most important ones. To enable additional measures and modify defaults, click Options under the Model screen.



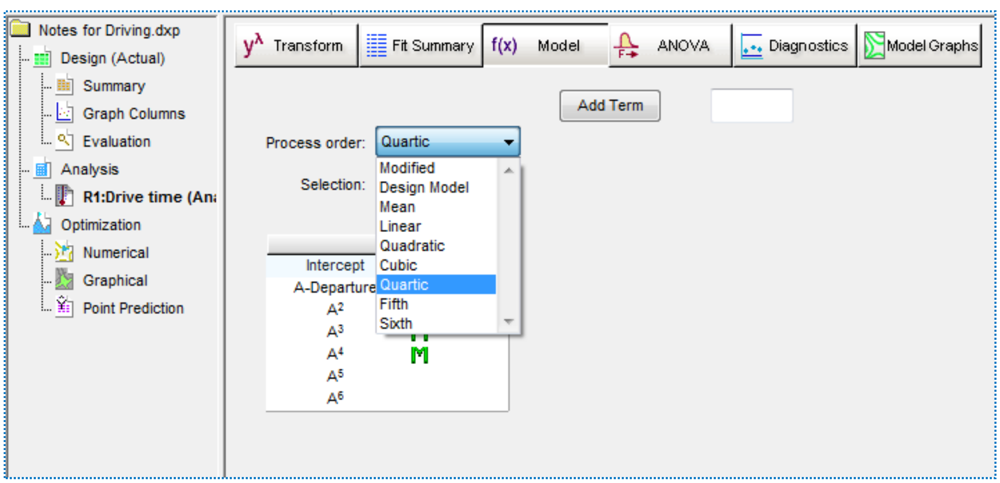
Evaluation options

Press ahead to the Graphs to see the plot of FDS – fraction of design space. Click the curve of standard error at a fraction near 0.8 (80 percent) to generate cross-reference lines like those shown in the screen shot below.



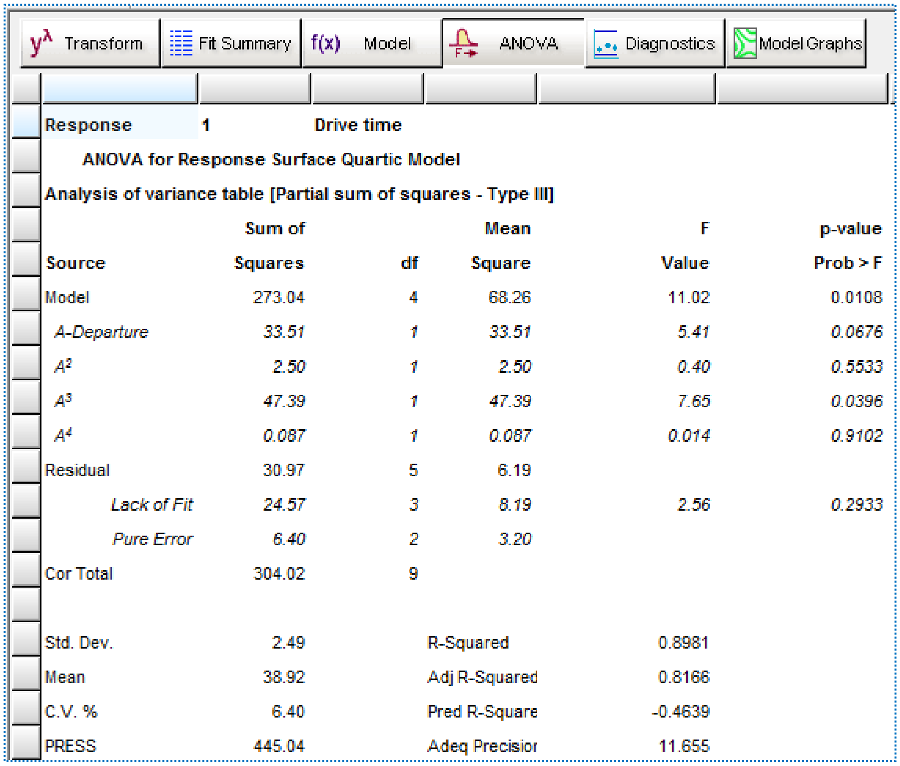
FDS graph

As noted in Screen Tips (hint: press the light-bulb icon), this is a line graph showing the relationship between the "volume" of the design space (area of interest) and amount of prediction error. The curve indicates what fraction (percentage) of the design space has a given prediction error or lower. In general, a lower and flatter FDS curve is better. The FDS graph provides very helpful information on scaled prediction variance (SPV) for comparing alternative test matrices – simple enough that even non-statisticians can see differences at a glance, and versatile for any type of experiment – mixture, process, or combined. For example, one could rerun the FDS graph for the cubic model and compare results and/or try some other experiment designs. However, let's not belabor the evaluation: Go back to the **Analysis** branch and click the **Drive time** node. Then press ahead to the ***Model*** and change **Process order** to **Quartic**.



Changing model to quartic for analysis

Now click the ***ANOVA*** button. Notice that not only does the A^4 term come out insignificant (p-value of 0.91), but the Pred R-Squared goes negative – not a good sign!



Response 1 Drive time

ANOVA for Response Surface Quartic Model

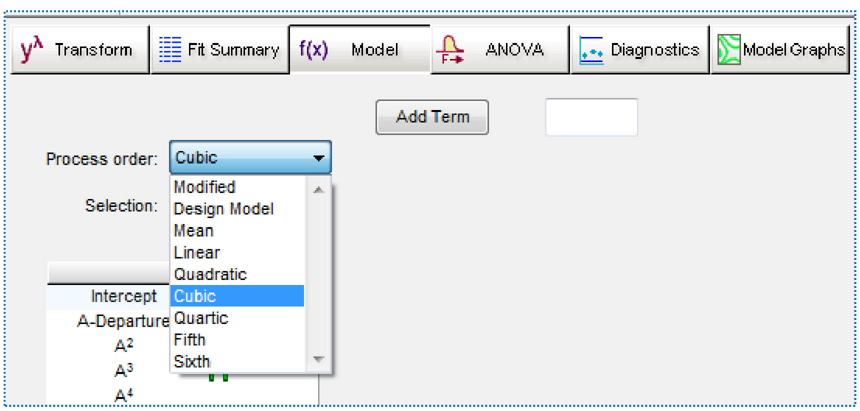
Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	273.04	4	68.26	11.02	0.0108
A-Departure	*33.51*	*1*	*33.51*	*5.41*	*0.0676*
A^2	*2.50*	*1*	*2.50*	*0.40*	*0.5533*
A^3	*47.39*	*1*	*47.39*	*7.65*	*0.0396*
A^4	*0.087*	*1*	*0.087*	*0.014*	*0.9102*
Residual	30.97	5	6.19		
Lack of Fit	*24.57*	*3*	*8.19*	*2.56*	*0.2933*
Pure Error	*6.40*	*2*	*3.20*		
Cor Total	304.02	9			

Std. Dev.	2.49	R-Squared	0.8981
Mean	38.92	Adj R-Squared	0.8166
C.V. %	6.40	Pred R-Square	-0.4639
PRESS	445.04	Adeq Precisior	11.655

ANOVA for quartic model (annotations turned of in View menu)

Before moving on to the next topic, return to the ***Model*** button and re-set the **Process order** to **Cubic**, which we recommend for this case.



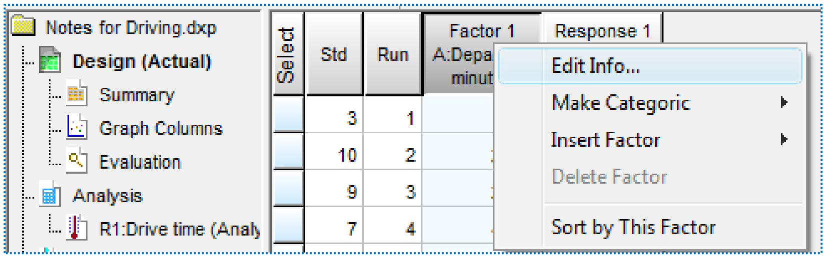
Back to the cubic model

By the way, Design-Expert distinguishes enough in this simplistic one-factor case to add up to sixth-order terms to the model list. However, in some cases, you may need to use the Add Term entry field. For example, in a two-factor RSM you can add terms such as A^2B^4 or A^3B^2.

Propagation of Error (POE)

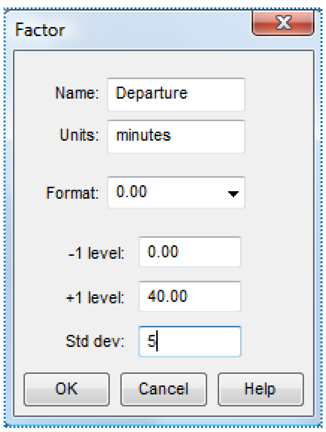
Seeing such a rapid increase in drive time predicted for late departures makes Mark more aware of how much the response depends on what time he leaves home. He realizes that a 5-minute deviation one way or the other would not be an unreasonable expectation. How will this cause the drive time to vary? Perhaps by aiming for a specific departure time, Mark might reduce drive-time variation caused by day-to-day differences when he leaves for work. Via its capability to calculate and plot propagation of error (POE), Design-Expert can provide enlightenment on these issues.

Click the **Design** branch to bring up the run-sheet (recipe procedure) for the experiment. Then right-click the column-header for **Factor 1 (**A:Departure) and select **Edit Info**.



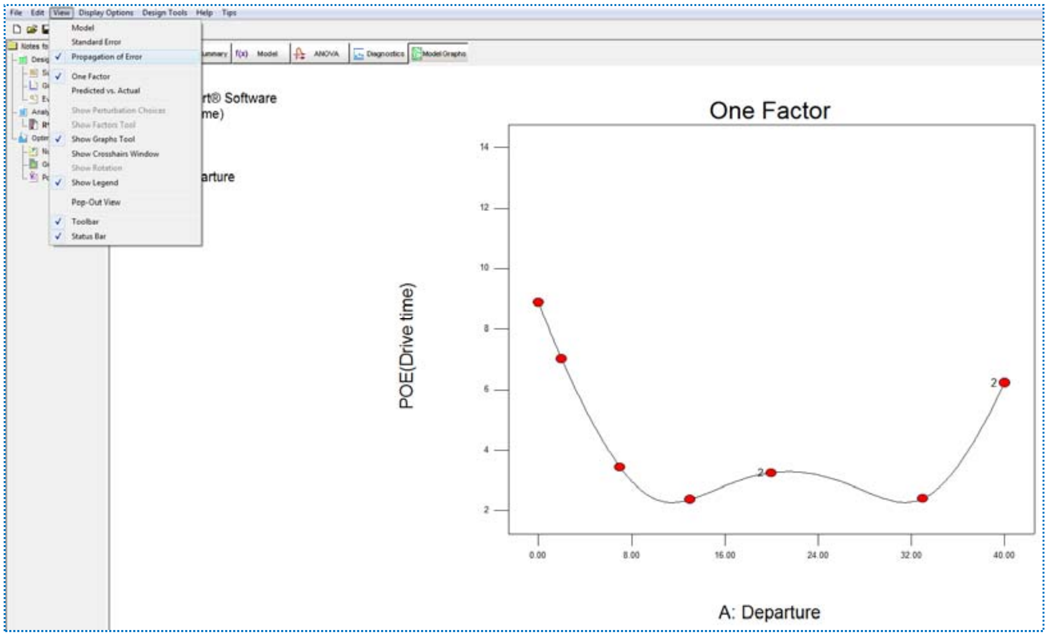
Editing info for the input factor

For **Std Dev** enter **5**.



Entering standard deviation for factor

Press **OK** and go back to the **Analysis** branch, click the **Drive time** node and go to ***Model Graphs***. Then from the **View** menu select **Propagation of Error**.



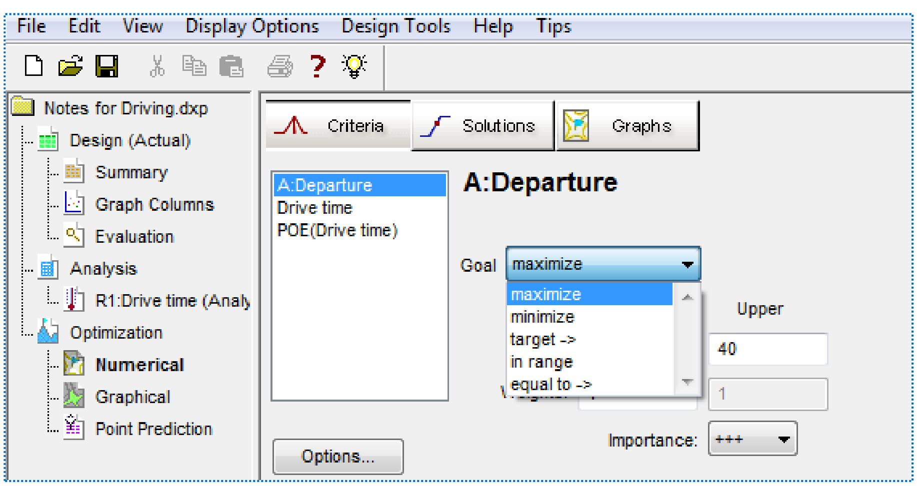
Plot for POE

Notice that POE is minimized at two times for departure, which correspond with flats on the wavy response plot you looked at earlier.

Multiple Response Optimization

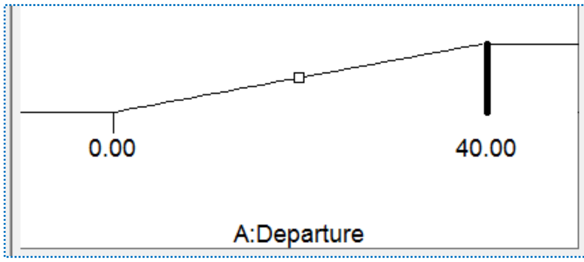
Ideally, Mark would like to leave as late as possible (to get more sleep every morning!) while minimizing his drive time – but making it the least variable. These goals can be established in Design-Expert software so it can look for the most desirable outcomes.

Under the **Optimization** branch, choose the **Numerical** node. For Departure, which comes up by default, click **Goal** and select **maximize**.



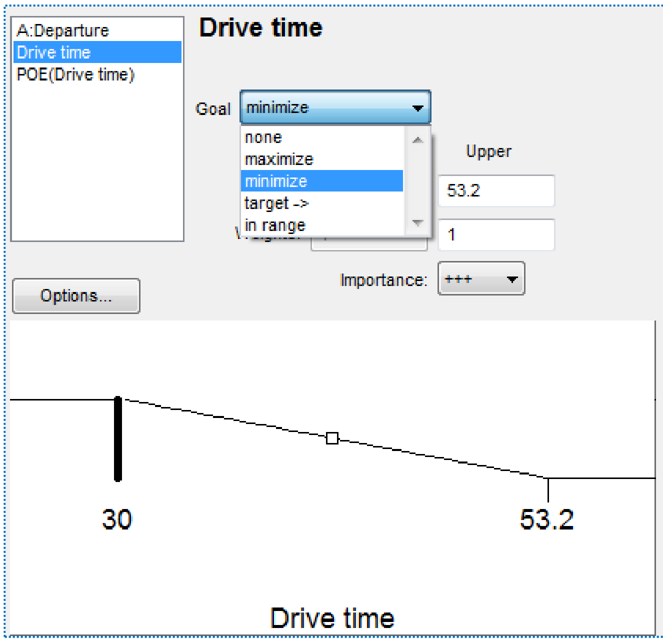
Setting goal for departure

The program pictures this goal as an upward ramp (/) to indicate that the higher this variable goes the more desirable it becomes.



Desirability ramp for departure – later the better (maximize)

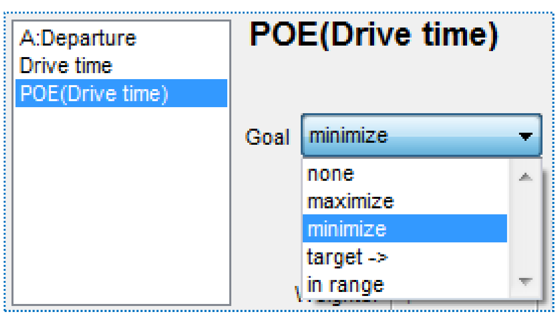
Next, click the response for **Drive time.** For its **Goal** select **minimize**.



Drive time minimized

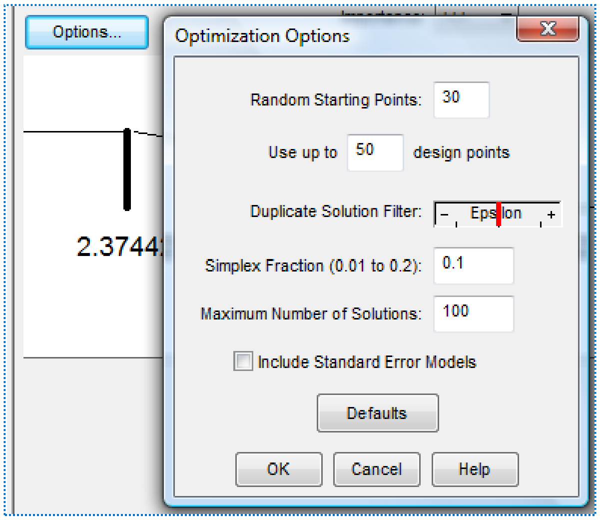
Notice the ramp now goes downward (\) to show that for this variable, lesser is better, that is, more desirable.

Lastly, to reduce variation in drive time caused by deviation in departure, click **POE (Drive Time)** and set its **Goal** to **minimize**.



Minimizing POE

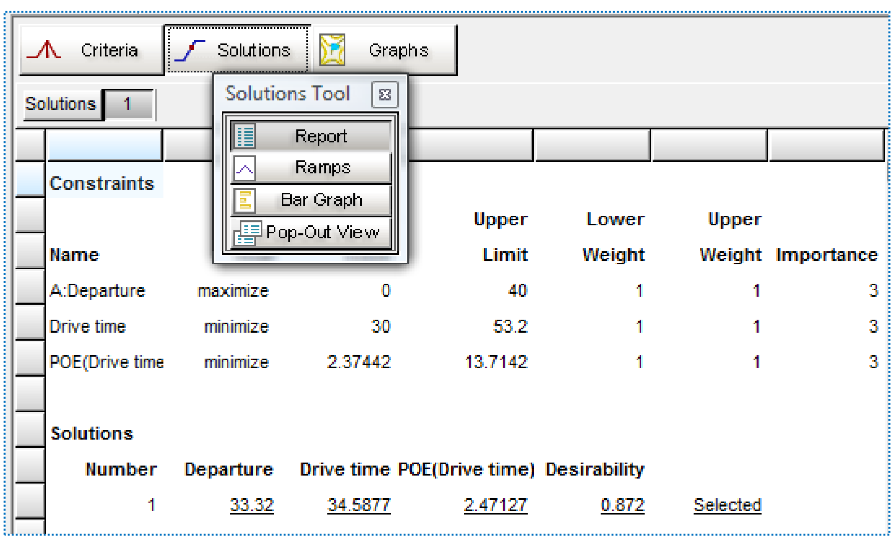
Before pressing ahead, click the **Options** button.



Options for numeric optimization

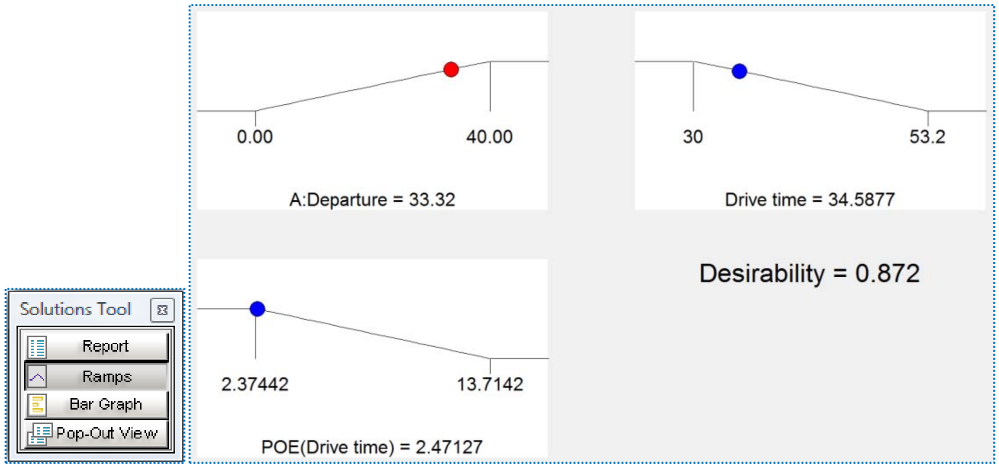
The settings here will affect the hill-climbing algorithm that Design-Expert uses to find the most desirable combination of variables. For details, check Help. Click **OK** to accept the defaults.

Press the ***Solutions*** button to see what Design-Expert recommends for the most desirable departure. The program now chooses a departure time at random and climbs up the desirability response surface. It repeats this process over and over, but in this case, the same point (within a value "epsilon" for the duplicate solution filter – see Optimization Options above) is found every time – a departure around 33 minutes beyond the earliest start acceptable by Mark for his morning commute. (Your result may vary somewhat due to the random starting points of the hill-climbing algorithm.)



Most desirable solution (your result may vary somewhat)

For a quick feel for optimal factor settings and the most desirable results, on the floating **Solutions Tool** click **Ramps**.



Ramps view of most desirable solution

Now Mark knows when it's best to leave for work while simultaneously maximizing the departure (and gaining more 'shut-eye'), minimizing his drive time, and minimizing propagation of error. The only thing that could possibly go wrong would be if all the other commuters learn how to use RSM and make use of Design-Expert. Mark hopes that none of you who are reading this tutorial live in his suburban neighborhood and work downtown.