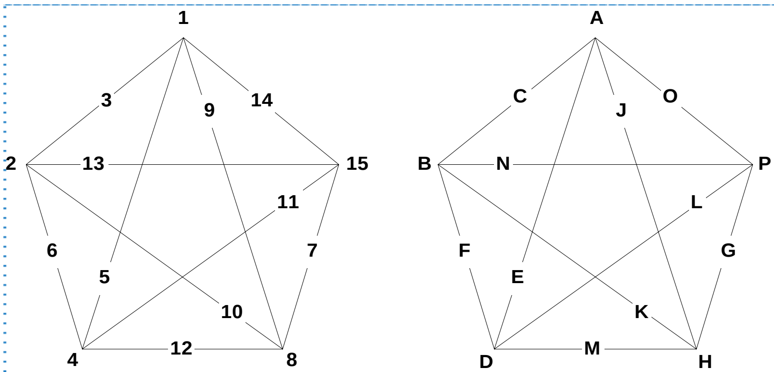
Taguchi Design Tutorial

Introduction

Taguchi's orthogonal arrays provide an alternative to standard factorial designs. Factors and interactions are assigned to the array columns via linear graphs. For example, look at the first of 18 linear graphs for the Taguchi L16 (16 run two-level factorial).



First linear graph for L16 array

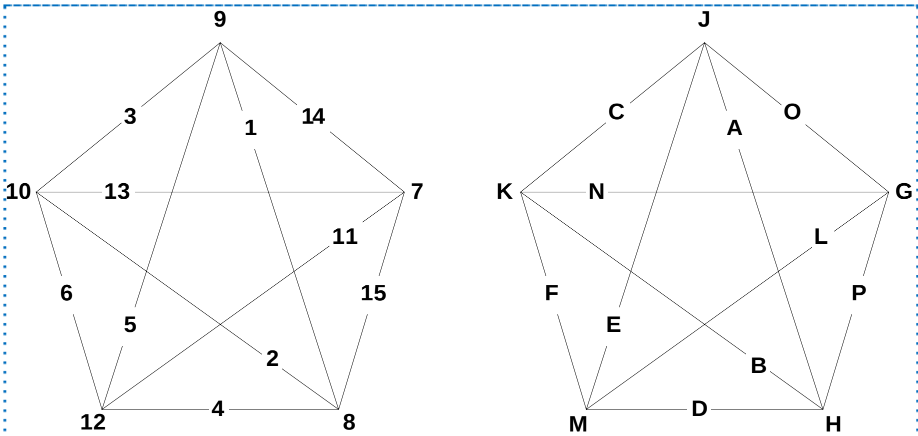
The figure at upper left displays 15 column numbers available for effect estimation. To the right you see the corresponding factor letters. Starting at the top and going counter-clockwise, you can see that factor C is connected to AB, implying confounding of factor C with the AB interaction. Factor F is connected to BD, and so forth. These relationships describe aliasing for a handful of the possible relationships. The complete alias structure for the L16, generated by Design-Expert® software, is shown below.

[A]	= A - BC - DE - FG - HJ -KL - MN - OP
[B]	= B - AC - DF - EG - HK -JL - MO - NP
[C]	= **C** - **AB** - DG - EF - HL - JK - MP - NO
[D]	= D - AE - BF - CG - HM - JN - KO - LP
[E]	= **E** - **AD** - BG - CF - HN - JM - KP - LO
[F]	= **F** - AG - **BD** - CE - HO - JP - KM - LN
[G]	= **G** - AF - BE - CD - **HP** - JO - KN - LM
[H]	= H - AJ - BK - CL - DM - EN - FO - GP
[J]	= **J** - **AH** - BL - CK - DN - EM - FP - GO
[K]	= **K** - AL - **BH** - CJ - DO - EP - FM - GN
[L]	= **L** - AK - BJ - CH - **DP** - EO - FN - GM
[M]	= **M** - AN - BO - CP - **DH** - EJ - FK - GL
[N]	= **N** - AM -**BP** - CO - DJ - EH - FL - GK
[O]	= **O** - **AP** - BM - CN - DK - EL - FH - GJ
[P]	= P - AO - BN - CM - DL -EK - FJ - GH

Aliasing of main effects with two-factor interactions for L16 (first linear graph)

The underlined effects are the aliases revealed by Taguchi's first linear graph.

The second of Taguchi's 18 linear graphs is given below.



Second linear graph for L16

This second linear graph reveals the underlined, italicized effects shown below.

[A] = ***A*** - BC - DE - FG - ***HJ*** - KL - MN - OP
[B] = ***B*** - AC - DF - EG - ***HK*** - JL - MO - NP
[C] = ***C*** - **AB** - DG - EF - HL - ***JK*** - MP - NO
[D] = ***D*** - AE - BF - CG - ***HM*** - JN - KO - LP
[E] = ***E*** - **AD** - BG - CF - HN - ***JM*** - KP - LO
[F] = ***F*** - AG - **BD** - CE - HO - JP - ***KM*** - LN
[G] = **G** - AF - BE - CD - **HP** - JO - KN - LM
[H] = H - AJ - BK - CL - DM - EN - FO - GP
[J] = **J** - **AH** - BL - CK - DN - EM - FP - GO
[K] = **K** - AL - **BH** - CJ - DO - EP - FM - GN
[L] = ***L*** - AK - BJ - CH - **DP** - EO - FN - ***GM***
[M] = **M** - AN - BO - CP - **DH** -EJ - FK - GL
[N] = ***N*** - AM - **BP** - CO - DJ - EH - FL - ***GK***
[O] = ***O*** - **AP** - BM - CN - DK - EL - FH - ***GJ***
[P] = ***P*** - AO - BN - CM - DL - EK - FJ - ***GH***

Aliasing of main effects with two-factor interactions for L16 (second linear graph)

In theory, you could build the entire alias structure by going through all 18 linear graphs. But why bother? The complete alias structure is given by Design-Expert software via its Design Evaluation tool.

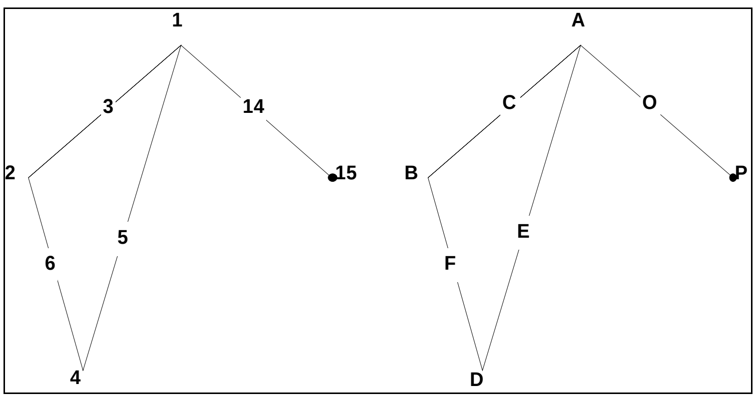
Case Study

To see how Design-Expert software handles Taguchi arrays, let's look at a welding example out of *System of Experiment Design*, Volume 1, page 189 (Quality Resources, 1991). The experimenters identified nine factors (see table below).

Factor	Units	Level 1	Level 2
Brand		J100	B17
Current	amps	150	130
Method		weaving	single
Drying		none	1 day
Thickness	mm	8	12
Angle	degrees	70	60
Stand-off	mm	1.5	3.0
Preheat		none	150 deg C
Material		SS41	SB35

Factors for welding experiment

The experimenters wanted estimates of several interactions: AB, AD, and BD. Looking at the first L16 linear graph (reproduced in part below), we see that column C can be used to estimate the AB interaction, column E to estimate AD, column F to estimate BD, and column O to estimate AP. Taguchi used columns M and N to estimate error.



Subset of first linear graph for L16 on welding

The factor assignments are summarized below.

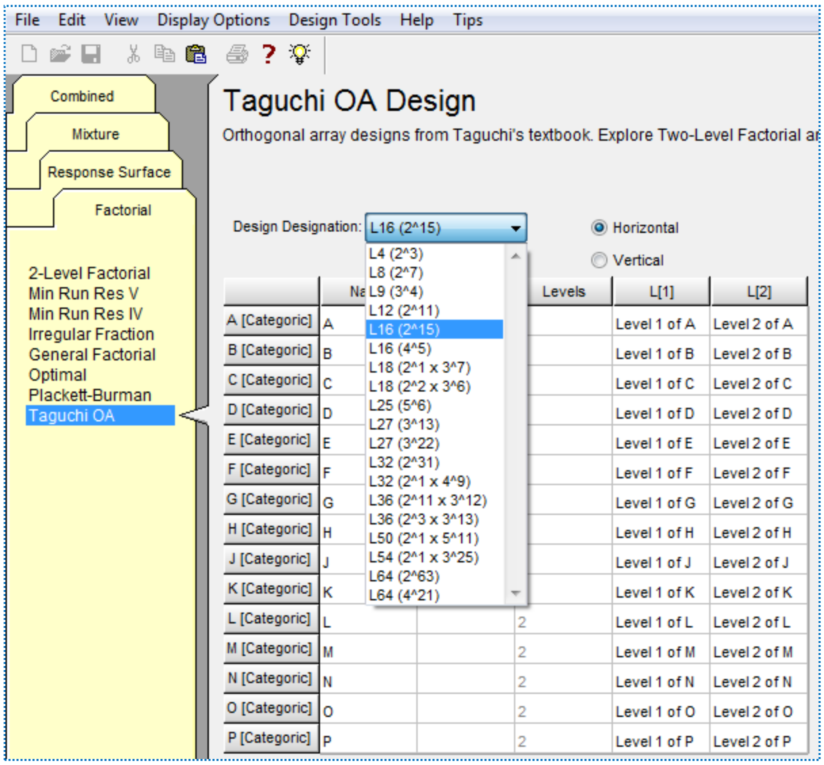
Column	Factor	Column	Factor
A	Brand		
B	Current	J	Angle
C	AB	K	Stand-off
D	Method	L	Preheat
E	AD	M	
F	BD	N	
G	Drying	O	AP
H	Thickness	P	Material

Factor assignments for L16 on welding

Columns M and N (blank) will be used to estimate error. (Note: there is no column labeled "I" in this or other designs because this is reserved for the intercept of the predictive model.)

Design the Experiment

Let's build this design. Choose **File**, **New Design** off the menu bar. (The blank-sheet icon 🗋 on the left of the toolbar is a quicker route to this screen. If you'd like to check this out, press Cancel to re-activate the tool bar. <u>But the fastest way, new to DX8, is to click New Design on the software's opening page.</u>) Then from the default **Factorial** tab click **Taguchi OA** (orthogonal array) and choose **L16(2^15)** from the pull down menu.



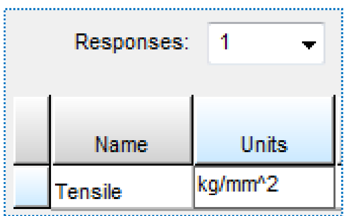
Selecting the Taguchi orthogonal array (OA)

Click the **Continue** button. The software then presents the alias structure for the chosen design. Notice that C is aliased with AB, E is aliased with AD, F is aliased with BD, and O is aliased with AP. Also, Design-Expert reserves the letter "I" for the intercept in predictive models, so it skips from factor "H" to "J."

[A] = A - BC - DE - FG - HJ - KL - MN - OP
[B] = B - AC - DF - EG - HK - JL - MO - NP
[C] = **C** - **AB** - DG - EF - HL - JK - MP - NO
[D] = D - AE - BF - CG - HM - JN - KO - LP
[E] = **E** - **AD** - BG - CF - HN - JM - KP - LO
[F] = **F** - AG - **BD** - CE - HO - JP - KM - LN
[G] = G - AF - BE - CD - HP - JO - KN - LM
[H] = H - AJ - BK - CL - DM - EN - FO - GP
[J] = J - AH - BL - CK - DN - EM - FP - GO
[K] = K - AL - BH - CJ - DO - EP - FM - GN
[L] = L - AK - BJ - CH - DP - EO - FN - GM
[M] = M - AN - BO - CP - DH - EJ - FK - GL
[N] = N - AM - BP - CO - DJ - EH - FL - GK
[O] = **O** - **AP** - BM - CN - DK - EL - FH - GJ
[P] = P - AO - BN - CM - DL - EK - FJ - GH

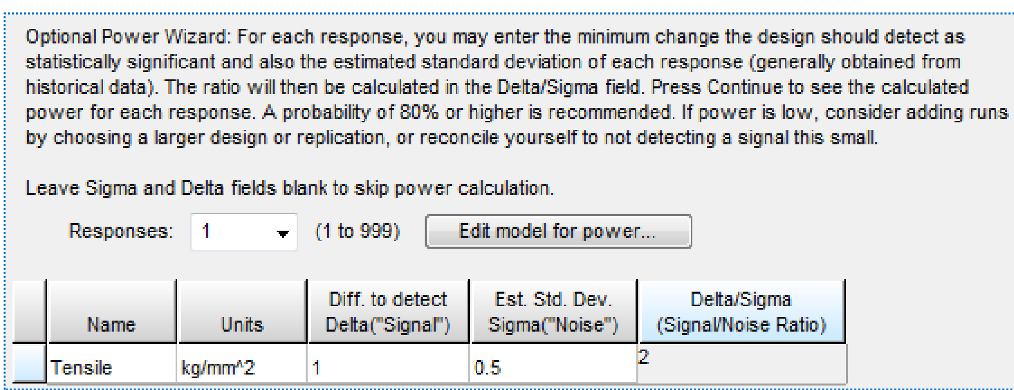
Alias structure for L16 two-level design (2^{15})

Click the **Continue** button. On all other designs you would now be prompted to enter factor names. However, for Taguchi designs this will be done later – after you generate the runs layout. Design-Expert now shows the response screen. Enter the 1 response name as "**Tensile**" and the units as "**kg/mm^2**".



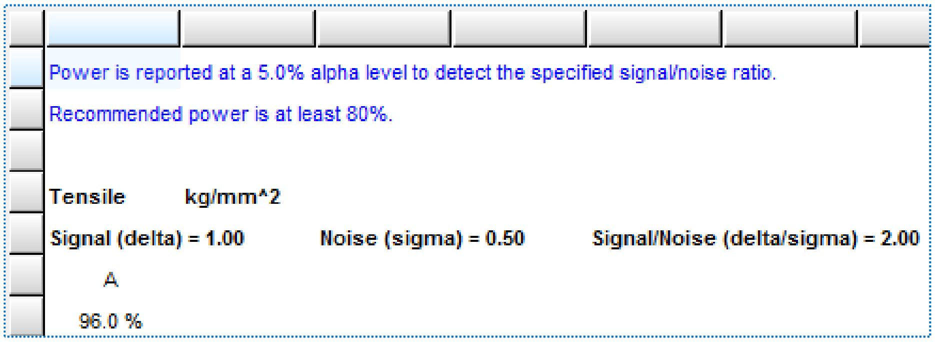
Response entry

At this point you can skip the remainder of the fields – used for calculating the power of your design – and continue on. However, it's best to gain an assessment of the power of this Taguchi design. Assume that it is beneficial to increase weld tensile strength by at least **1** unit on average, and that quality control data generates a standard deviation of **0.5**. Enter these values as shown below so Design-Expert can compute the signal to noise ratio – for this design: **2**.



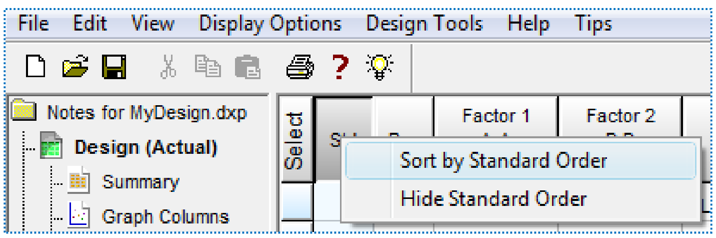
Optional power wizard – necessary inputs entered

Press **Continue** to see the calculated power, which in this case far exceeds the recommended level of 80 percent – the probability of seeing the desired difference in one main effect (an assumption made by Design-Expert for resolution III designs like this).



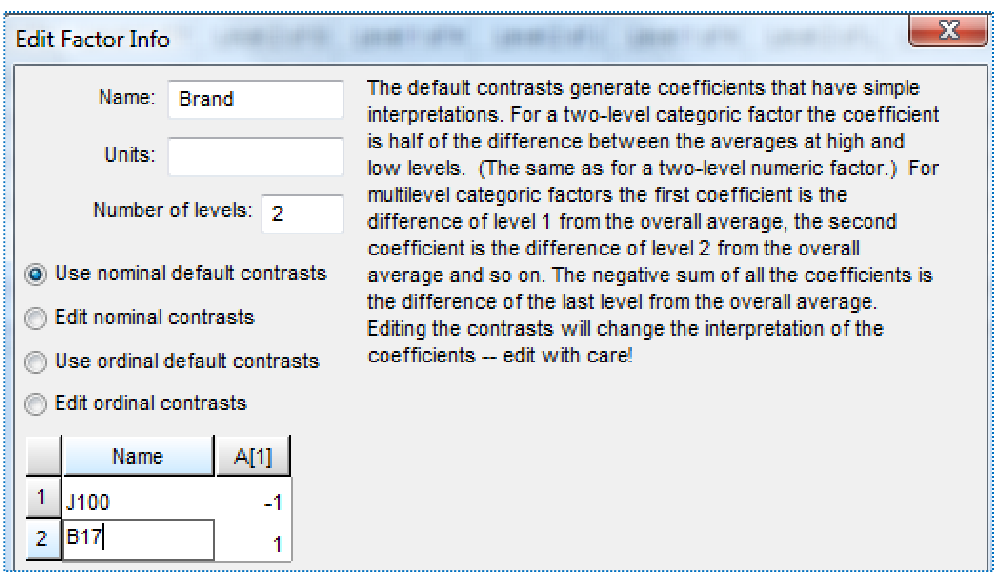
Results of power calculation

Click **Continue** to accept these inputs and generate the design layout window. Design-Expert now displays the experimental runs in random order. <u>Right click</u> the **Std** column and choose **Sort by Standard Order** to see Taguchi's design order.



Sorting design by standard order

Next, <u>right click</u> the factor **A** heading and choose **Edit Info**: Enter "**Brand**" as the name and "**J100**" and "**B17**" as level 1 and level 2. These are nominal (named) contrasts, so leave that option as the default. Click **OK**.

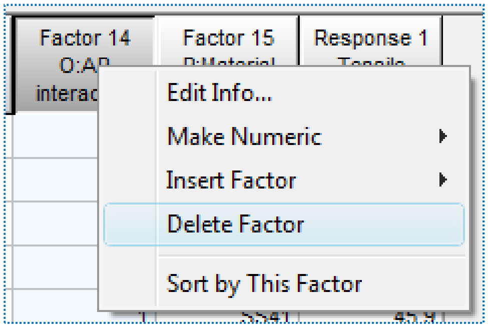


Using edit factor info screen

The remaining factors can be entered in the same way, but to save time, read in the response data via **File**, **Open Design** from the main menu. Select the file named **Taguchi-L16.dxp**. (If you're asked to save MyDesign.dez, click No.)

Recall that several of the columns (C, E, F, and O) are being used to estimate interactions. Two others (M and N) are being used to estimate error. We can delete all these columns and fit the interactions directly. This greatly simplifies the analysis.

To do this, right click the header for column **O** (Factor 14) to bring up the menu shown below. Choose **Delete Factor** and confirm with a **Yes**.



Right-click menu for factor column in design layout

When deleting factors, the letters associated with the remaining factors change, so to avoid confusion, always start at the right and work left. All this work will eventually pay off by putting you in position to take advantage of Design-Expert's powerful analytical capabilities.

Repeat the **Delete Factor** operation on the columns for factors **N**, **M**, **F**, **E** and **C**. When you finish deleting columns, only nine experimental factors should remain. Right click the **Std** column and **Sort by Standard Order**. Your design should now look like that pictured below.

Select	Std	Run	Factor 1 A:Brand	Factor 2 B:Current amps	Factor 3 C:Method	Factor 4 D:Drying	Factor 5 E:Thickness mm	Factor 6 F:Angle degrees	Factor 7 G:Stand-off mm	Factor 8 H:Preheat	Factor 9 J:Material	Response 1 Tensile kg/mm^2
	1	10	J100	150	weaving	none	8	70	1.5	none	SS41	43.7
	2	16	J100	150	weaving	none	12	60	6.0	150 deg C	SB35	40.2
	3	4	J100	150	single	1 day	8	70	1.5	none	SB35	42.4
	4	13	J100	150	single	1 day	12	60	6.0	150 deg C	SS41	44.7
	5	6	J100	130	weaving	1 day	8	70	6.0	150 deg C	SB35	42.4
	6	5	J100	130	weaving	1 day	12	60	1.5	none	SS41	45.9
	7	3	J100	130	single	none	8	70	6.0	150 deg C	SS41	42.2
	8	2	J100	130	single	none	12	60	1.5	none	SB35	40.6
	9	1	B17	150	weaving	1 day	8	60	1.5	150 deg C	SB35	42.4
	10	12	B17	150	weaving	1 day	12	70	6.0	none	SS41	45.5
	11	7	B17	150	single	none	8	60	1.5	150 deg C	SS41	43.6
	12	8	B17	150	single	none	12	70	6.0	none	SB35	40.6
	13	14	B17	130	weaving	none	8	60	6.0	none	SS41	44
	14	11	B17	130	weaving	none	12	70	1.5	150 deg C	SB35	40.2
	15	9	B17	130	single	1 day	8	60	6.0	none	SB35	42.5
	16	15	B17	130	single	1 day	12	70	1.5	150 deg C	SS41	46.5

Taguchi L16 after deleting columns so only 9 factors remain

It is helpful to make a table for cross-referencing the original assignment of factor letters with the new (condensed) list.

Original (9)	New (9)	Discarded (6)
A: Brand	**A: Brand**	K: Stand-off
B: Current	**B: Current**	L: Preheat
C: AB	**C: Method**	M: error
D: Method	**D: Drying**	N: error
E: AD	**E: Thickness**	O: AP
F: BD	**F: Angle**	P: Material
G: Drying	**G: Stand-off**	
H: Thickness	**H: Preheat**	
J: Angle	**J: Material**	

Cross-reference table for factor letter assignments

Note that some interactions also get re-labeled as shown in the table below: AB stays **AB**, but AD becomes **AC**; BD becomes **BC** and AP becomes **AJ**.

To review alias structure, click the design **Evaluation** node. For **Order** select **2FI** (two-factor interaction) and click ***Results***. In the related table below, we ignored interactions of three or more factors and underlined two-factor interactions of interest.

```
Factorial Effects Aliases
[Est. Terms]    Aliased Terms
   [Intercept] = Intercept
   [A] = A - EF - GH
   [B] = B - EG - FH
   [C] = C - HJ
   [D] = D - EJ
   [E] = E - AF - BG - DJ
   [F] = F - AE - BH
   [G] = G - AH - BE
   [H] = H - AG - BF - CJ
   [J] = J - CH - DE
   [AB] = AB + CD + EH + FG
   [AC] = AC + BD + GJ
   [AD] = AD + BC + FJ
   [AJ] = AJ + CG + DF
   [BJ] = BJ + CF + DG
   [CE] = CE + DH
```

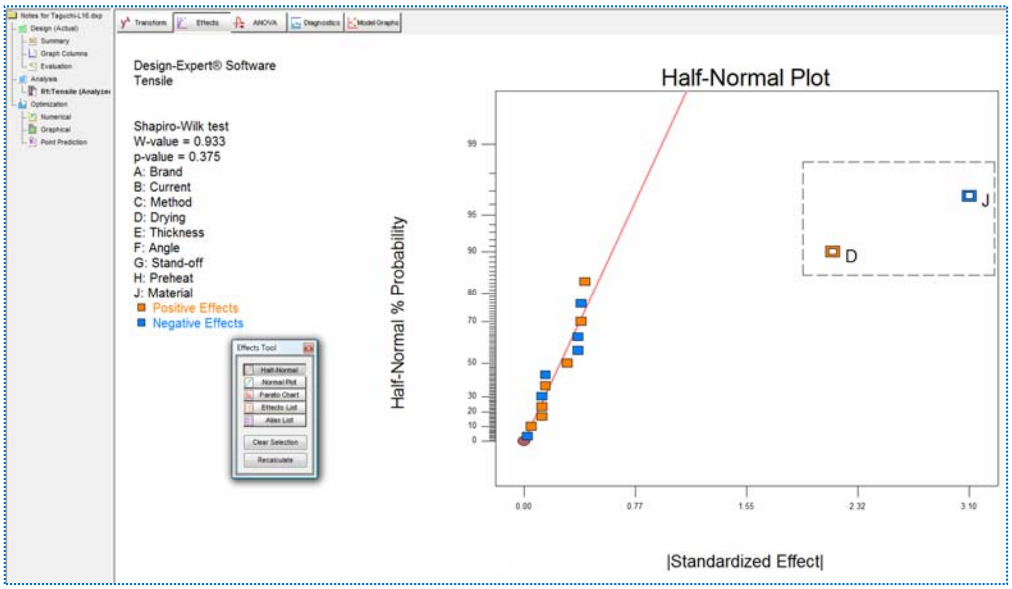
Alias structure after deleting columns from L16

Notice that all the main effects, plus the four interactions of interest, are aliased with one or more two-factor interactions. The effects now labeled BJ and CE are the two columns used to estimate error, but they too are aliased with two-factor interactions. <u>All</u> of these aliased interactions must be negligible for an accurate analysis.

Analyze the Results

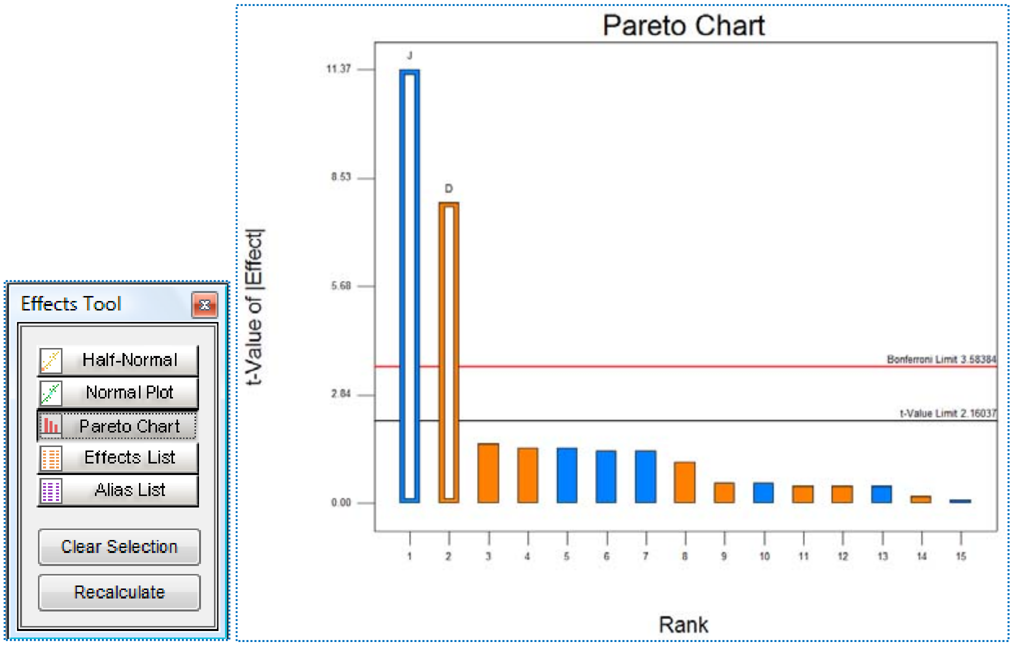
To analyze the results, follow the usual procedure for two-level factorials as demonstrated earlier in the Factorial Design Tutorials. (If you haven't already done so, go back and complete this tutorial.)

Click the analysis node labeled **Tensile**, which is found in the tree structure along the left of the main window. Then, click the ***Effects*** button displayed in the toolbar at the top of the main window. Click the two largest effects (**J** and **D**) on the half-normal plot of effects, or rope them off as shown below.



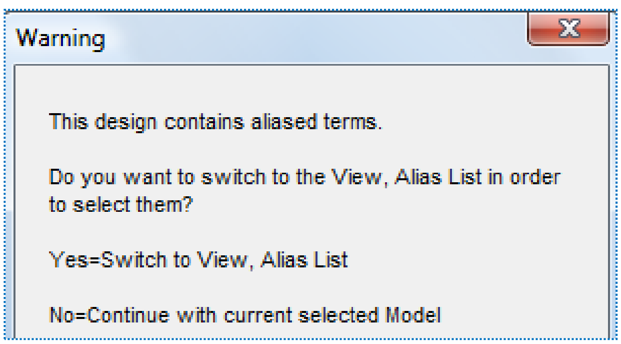
Half-normal plot of effects

On the floating **Effects Tool** press **Pareto Chart** for another view on the relative magnitude of effects. It's very clear now that factors J and D stand out.



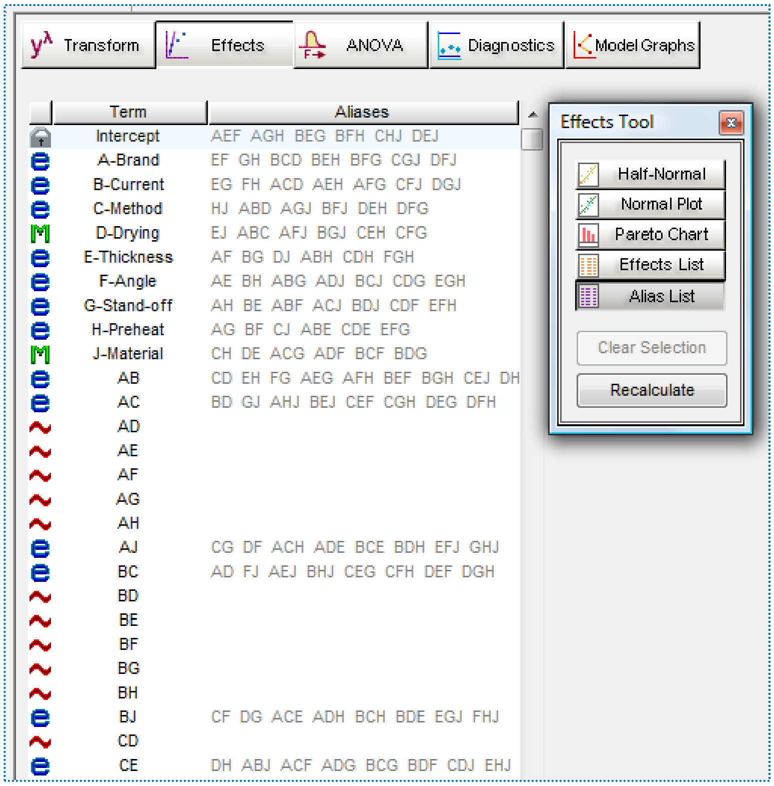
Pareto chart of effects

Click the ***ANOVA*** button. Design-Expert now warns you about aliasing and offers a list for you to view.



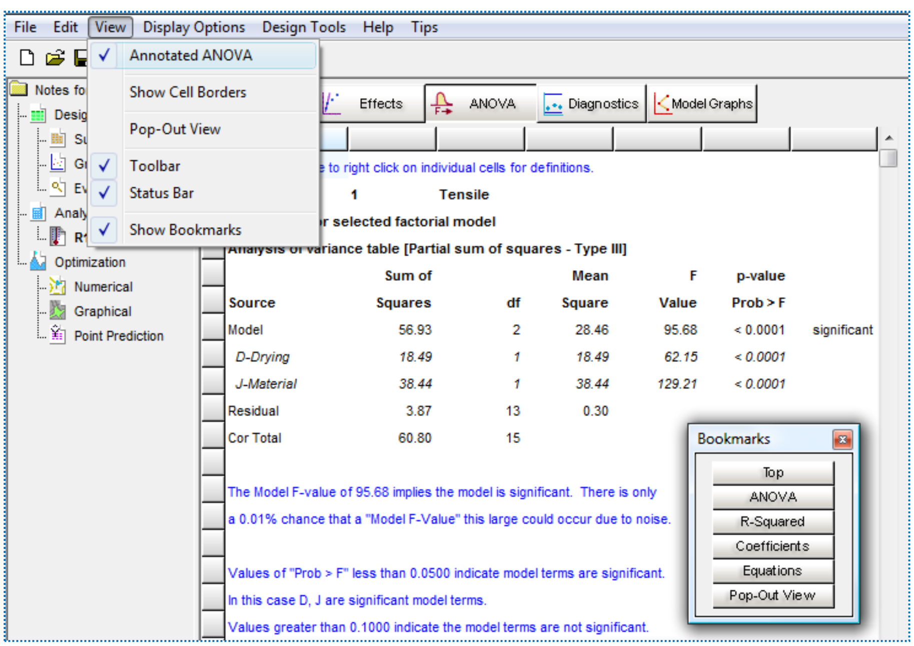
Warning that design contains aliased terms

Click **Yes** to see the aliases again – this time with modeled terms identified by "M" and the others labeled "e" for error.



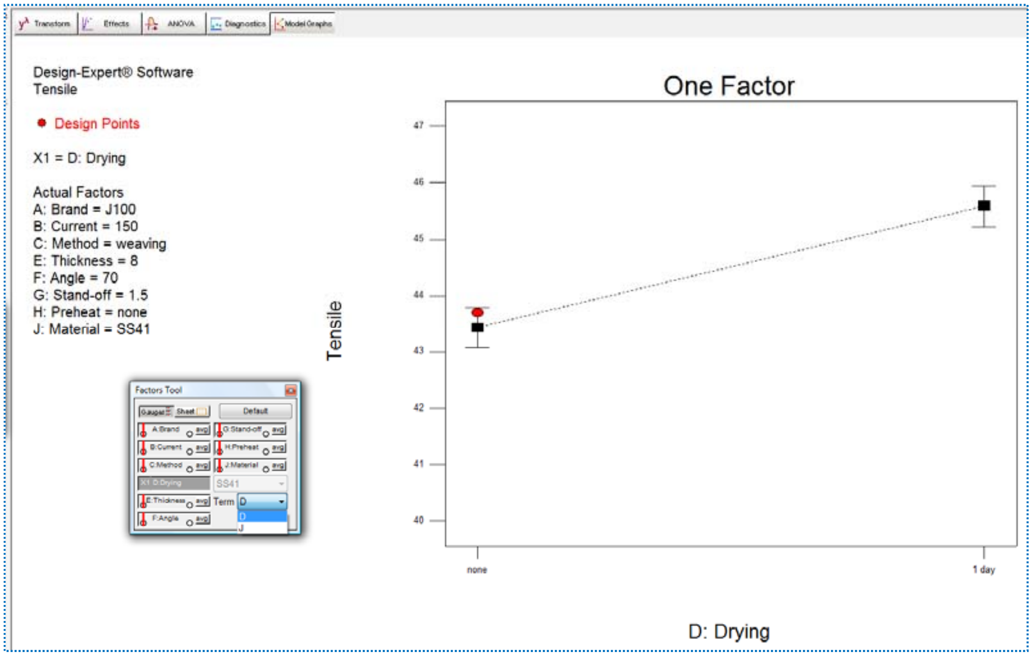
View of alias list

Again click ***ANOVA***. You should now see an annotated ANOVA report by default. If not, select View, Annotated ANOVA. The ANOVA confirms that the effects of D and J are statistically significant.



ANOVA report

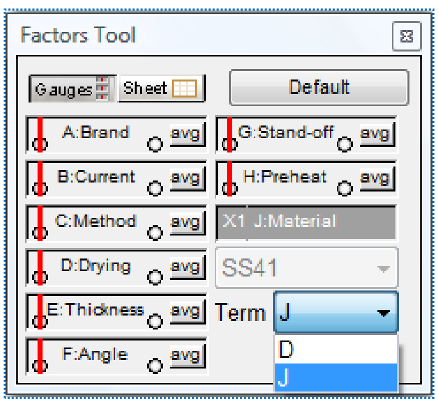
Scroll down, or use the handy bookmarks, for more statistics. Then click ahead to the ***Diagnostics***. The diagnostic results don't look great – for example, the normal plot of residuals looks a bit off-kilter –but they're acceptable. Click the ***Model Graphs*** button and, if not already picked by default, select factor **D** as the **Term** on the floating factors tool. The following graph should now appear.



One-factor plot of the main effect of factor D (drying)

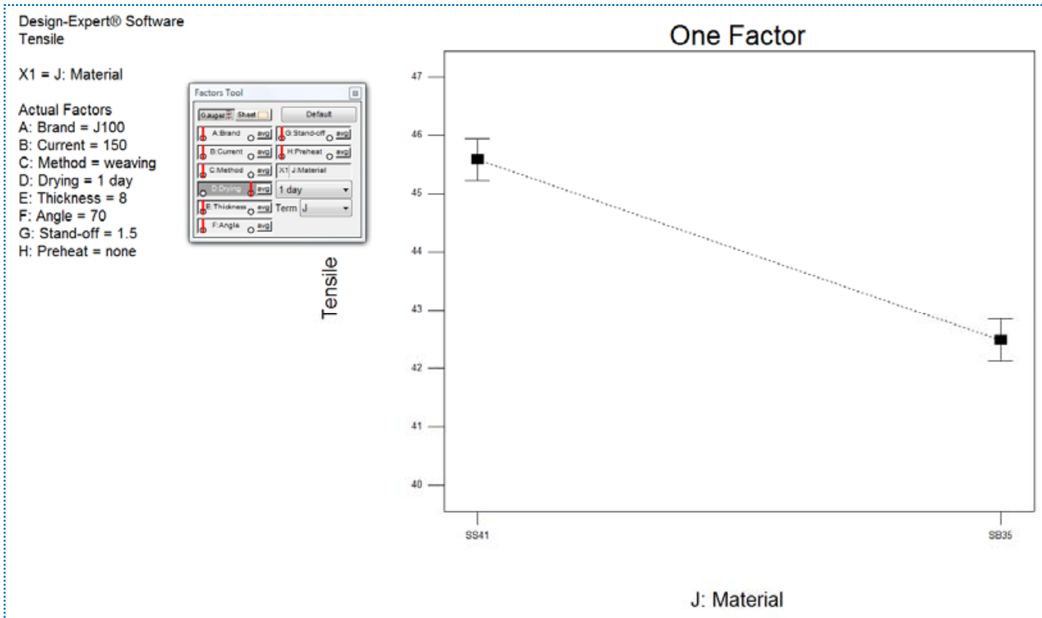
You may be wondering about the circular red symbol. As indicated by the legend to the left of the screen in red, this is an actual design point. Due to the fractional nature of Taguchi designs, you won't see too many points on the plots of predicted effects. Click any point(s) that do appear to see readouts of their actual response value.

On the floating **Factors Tool**, click the **Term** down arrow now and select **J** (or right-click on the J bar to make it the X1-axis). Notice that Design-Expert defaults to the "lower" level of the categorical factors.



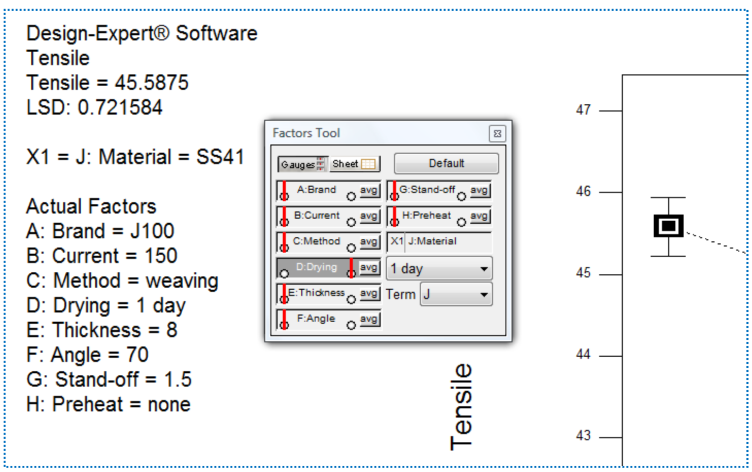
Plotting the second significant main effect (J)

On the factors tool, <u>click</u> the "upper" level (far right point) of **D:Drying**, which we now know is best for tensile strength. This shifts the red line to the far right, as shown below.



Plot of main effect J (material) with factor D set at high level

The square symbols at either end of the lines show the predicted outcomes. The vertical I-beam-shaped bars appearing through these points represent the least significant difference (LSD) at a 95% confidence level (the default). <u>Click the square symbol (predicted value) at the upper left</u> to get a readout on the LSD (it appears to the left of the graph). Note that this LSD does not overlap through an imaginary horizontal line with the LSD at the lower right. This provides visual verification that the effect shown on the plot is statistically significant.



Maximal point clicked with predicted response noted and LSD reported

This concludes our tutorial on Taguchi orthogonal arrays. Use these designs with caution! Take advantage of Design-Expert's design evaluation feature for examining aliases. Make sure that likely interactions are not confounded with main effects. For example, in this case, only two main effects (D and J) appear to be significant, but as shown in the alias table, perhaps D is really EJ, and J could be CH and/or DE. Therefore, it would be a good idea to do a follow-up experiment to confirm the effects of D and J.