Two-Level Factorial Split Plot Tutorial

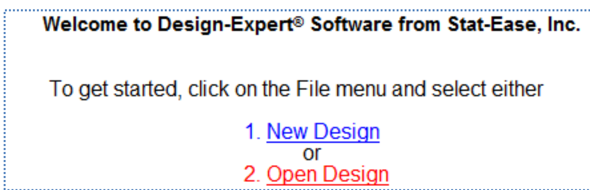
Introduction

Very often, experimenters set up two-level factorial designs with the best intentions of running them in random order, but they find that a given factor, such as temperature, cannot be easily changed. This creates a design structure called "split plot," which necessitates that users of Design-Expert® software specify how model terms be treated in the analysis of variance (ANOVA). For an example, see the "Split-Plot General Factorial" tutorial provided earlier.

For this special case of a two-level factorial design, the handy half-normal plot for effect selection can be adapted to deal with the split plot structure. This will be illustrated very briefly via a case study. After completing this tutorial, take a look at a slightly more complex experiment detailed in "Strip Block Design Gives Battery Experiment a Charge" in the July 2005 issue of *Stat-Teaser* newsletter posted by Stat-Ease at www.statease.com/newsltr.html.

In all cases, by properly accounting for the variances from split plot structures, the experimenter gains a more accurate assessment of significance for specific effects.

From the Welcome screen in Design-Expert click **Open Design**.



Opening the design

Then double-click on **Plasma.dxp**. These data, shown on the following page, come from an experiment on a plasma treatment process aimed at making paper more receptive to ink (Box, Bisgaard, et al., "Quality Quandries: Two-Level Factorials Run as Split Plot Experiments," *Quality Engineering*, 8(4), 705-708 (1996)). Notice there are five factors in 32 runs – a full two-level factorial (2^5). If you look carefully at the pattern of highs and lows, you see that Factor E (paper type) is not randomized.

To save time, the experimenters set up their plasma reactor at the conditions specified by factors A through D (randomized), and then processed the two paper types (E) together. (The actual placement of paper in the reactor, right versus left, was randomized by a flip of a coin.) This forms a split plot design, broken down as follows:

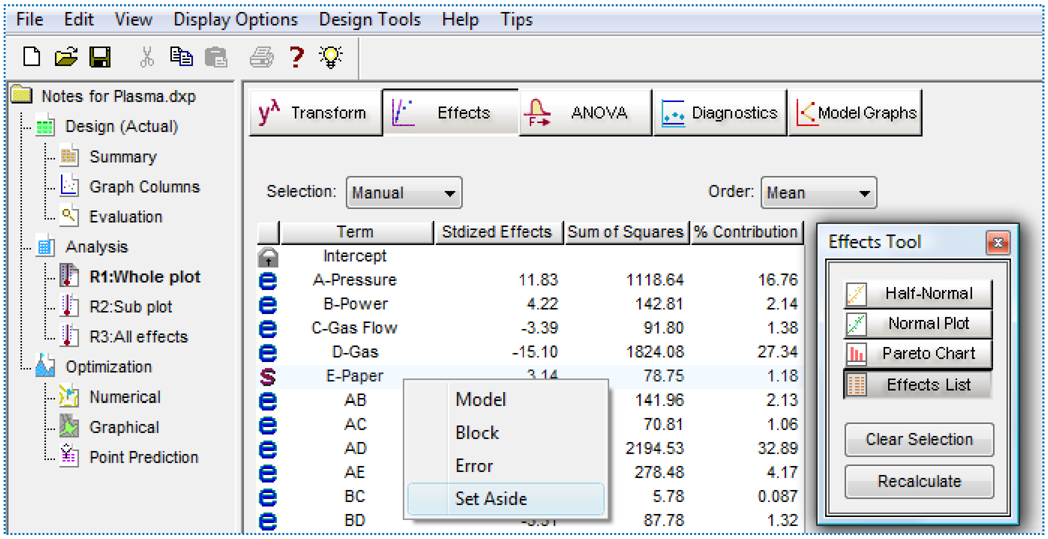
- Whole-plot factors: A through D (and associated interactions)
- Subplot factor: E (and any interactions involving this factor)

The trick is to keep these groups separate for the analysis of variance, because the residual errors differ. Design-Expert makes this relatively easy, as you will see next. To keep the differing analyses separated, the responses have been copied into three columns labeled "Whole plot," "Sub plot," and "All effects."

Run	A: Pressure	B: Power	C: Gas Flow	D: Gas Type	E: Paper Type	Contact angle
1	-1	-1	1	Oxygen	E1	37.6
2	-1	-1	1	Oxygen	E2	43.5
3	1	-1	-1	Oxygen	E1	41.2
4	1	-1	-1	Oxygen	E2	38.2
5	1	-1	-1	SiCl4	E1	56.8
6	1	-1	-1	SiCl4	E2	56.2
7	1	-1	1	SiCl4	E1	47.5
8	1	-1	1	SiCl4	E2	43.2
9	-1	1	-1	SiCl4	E1	25.6
10	-1	1	-1	SiCl4	E2	33
11	-1	1	-1	Oxygen	E1	55.8
12	-1	1	-1	Oxygen	E2	62.9
13	-1	-1	1	SiCl4	E1	13.3
14	-1	-1	1	SiCl4	E2	23.7
15	1	-1	1	Oxygen	E1	47.2
16	1	-1	1	Oxygen	E2	44.8
17	1	1	1	SiCl4	E1	49.5
18	1	1	1	SiCl4	E2	48.2
19	-1	-1	-1	SiCl4	E1	5
20	-1	-1	-1	SiCl4	E2	18.1
21	-1	1	1	SiCl4	E1	11.3
22	-1	1	1	SiCl4	E2	23.9
23	-1	-1	-1	Oxygen	E1	48.6
24	-1	-1	-1	Oxygen	E2	57
25	1	1	1	Oxygen	E1	48.7
26	1	1	1	Oxygen	E2	44.4
27	-1	1	1	Oxygen	E1	47.2
28	-1	1	1	Oxygen	E2	54.6
29	1	1	-1	Oxygen	E1	53.5
30	1	1	-1	Oxygen	E2	51.3
31	1	1	-1	SiCl4	E1	41.8
32	1	1	-1	SiCl4	E2	37.8

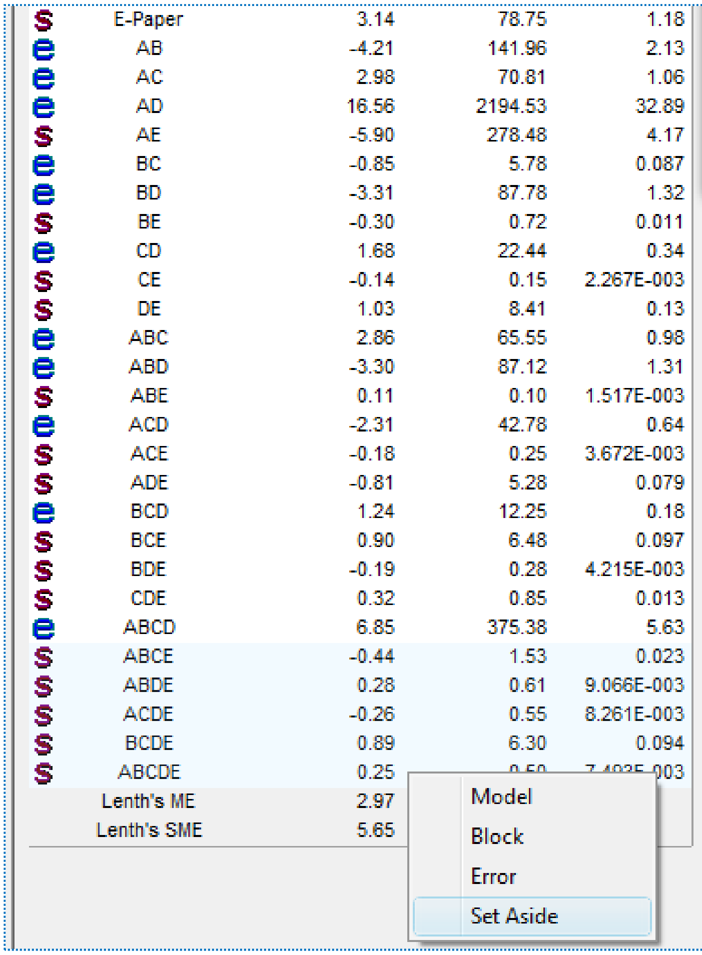
Two-level factorial run as split plot – the raw data

Start your analysis by clicking the response node **Whole plot** and press the ***Effects*** button. On the floating **Effects Tool** press **Effects List** (or from the main menu, select View, Effects List). Right-click factor **E** and choose **Set Aside** as shown below.



Setting aside term E as an effect

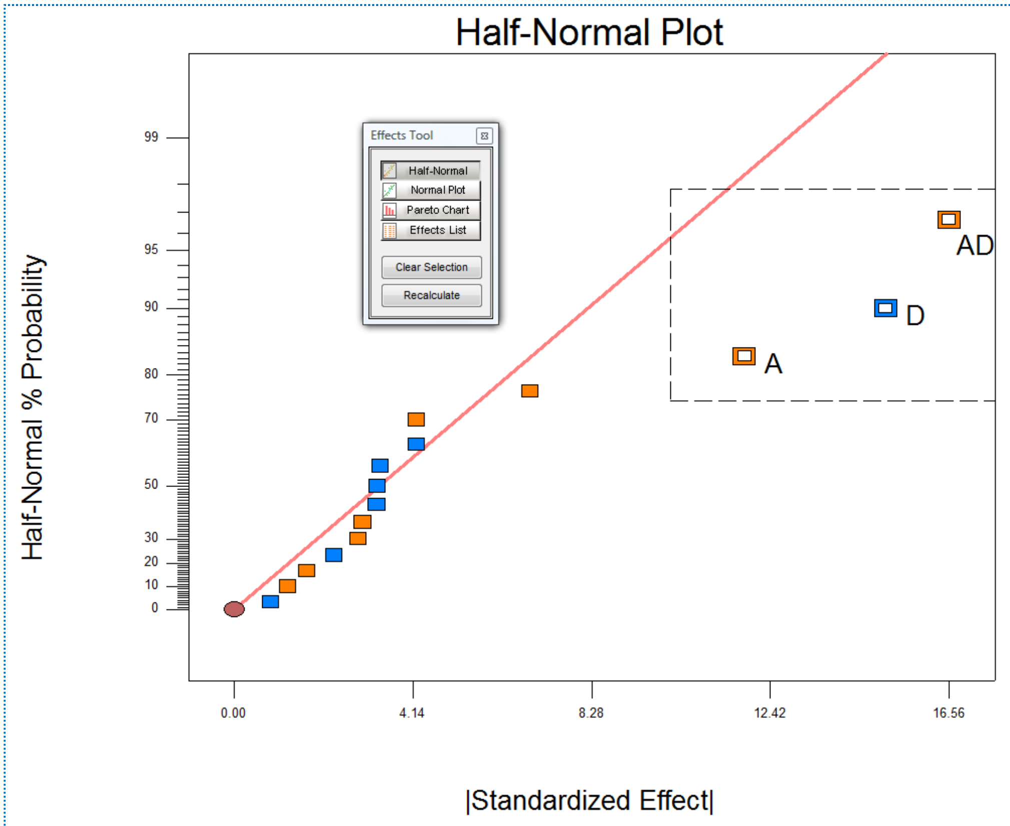
Do the same (set aside or "s") for all interaction terms involving E. (Suggestion: drag over the last few blocks of E terms to highlight them, and then right-click to change all of them to "set aside.") Your interaction terms should now match those shown below.



S	E-Paper	3.14	78.75	1.18
e	AB	-4.21	141.96	2.13
e	AC	2.98	70.81	1.06
e	AD	16.56	2194.53	32.89
S	AE	-5.90	278.48	4.17
e	BC	-0.85	5.78	0.087
e	BD	-3.31	87.78	1.32
S	BE	-0.30	0.72	0.011
e	CD	1.68	22.44	0.34
S	CE	-0.14	0.15	2.267E-003
S	DE	1.03	8.41	0.13
e	ABC	2.86	65.55	0.98
e	ABD	-3.30	87.12	1.31
S	ABE	0.11	0.10	1.517E-003
e	ACD	-2.31	42.78	0.64
S	ACE	-0.18	0.25	3.672E-003
S	ADE	-0.81	5.28	0.079
e	BCD	1.24	12.25	0.18
S	BCE	0.90	6.48	0.097
S	BDE	-0.19	0.28	4.215E-003
S	CDE	0.32	0.85	0.013
e	ABCD	6.85	375.38	5.63
S	ABCE	-0.44	1.53	0.023
S	ABDE	0.28	0.61	9.066E-003
S	ACDE	-0.26	0.55	8.261E-003
S	BCDE	0.89	6.30	0.094
S	ABCDE	0.25	[illegible]	[illegible]
	Lenth's ME	2.97		
	Lenth's SME	5.65		

Modeling whole-plot factors and associated interaction effects

Now, on the **Effects Tool** press **Half-Normal**. The effects of A, D, and interaction AD stand out. Click or rope off these effects as shown below. (Do not pick the next biggest effect because it is the four-factor interaction ABCD, which makes no sense as a practical matter.)



Half-normal plot of whole-plot and associated interaction effects

Click the ***ANOVA*** button to verify the significance of these whole-plot effects – on the floating **Bookmarks** tool press **ANOVA** to see the p-values.

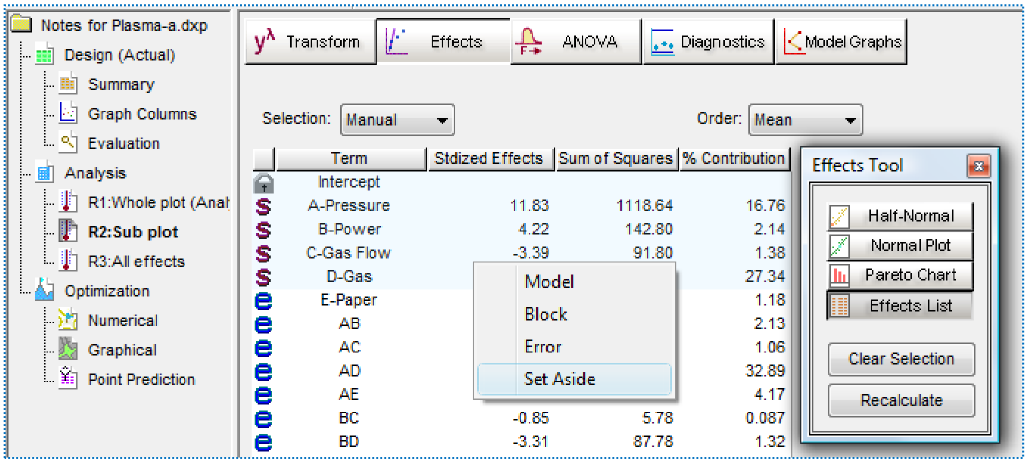
Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	5137.26	3	1712.42	17.92	< 0.0001	significant
A-Pressu	*1118.64*	*1*	*1118.64*	*11.71*	*0.0051*	
D-Gas	*1824.08*	*1*	*1824.08*	*19.09*	*0.0009*	
AD	*2194.53*	*1*	*2194.53*	*22.97*	*0.0004*	
Residual	1146.46	12	95.54			
Cor Total	6283.72	15				

ANOVA for whole plot model

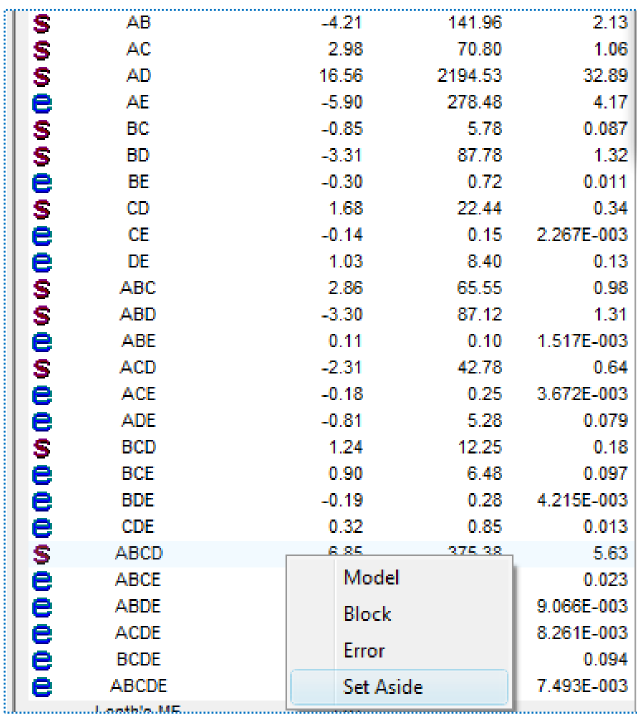
Do not bother looking at the Diagnostics or Model Graphs because the model is missing the subplot effects. The last step in the analysis will be to combine all the whole-plot and subplot terms into one overall model. Only then can you obtain a proper view of diagnostic and model graphs. Preserve your work thus far by selecting **File**, **Save As** and modifying the name to **Plasma-a.dxp** (or anything else you'd like that will leave the original tutorial file as-is).

Let's move on to investigating the subplot effects. Click the **Sub plot** node for analysis. Press the ***Effects*** button and the **Effects List** option on the floating tool. Drag over whole plot terms **A**, **B**, **C** and **D** to highlight them, then right-click and choose **Set Aside** ("s").



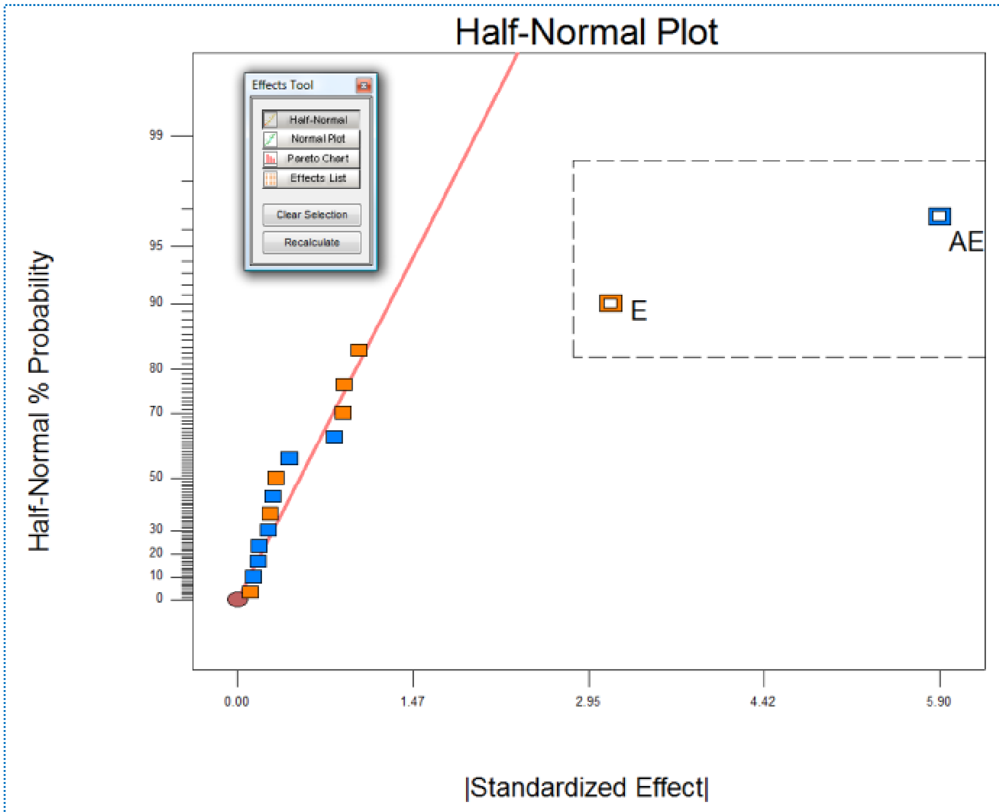
Modeling subplot factor E and setting aside the other (whole plot) factors

Right click all other terms NOT including letter E and change them to **Set Aside**. (Alternate approach to reduce mouse clicks: Drag over a block of terms to highlight them, then right-click and select Set Aside.) Term E and all interactions involving this factor should be left at their default settings of e for error.



Setting aside interaction terms not involved with subplot factor E

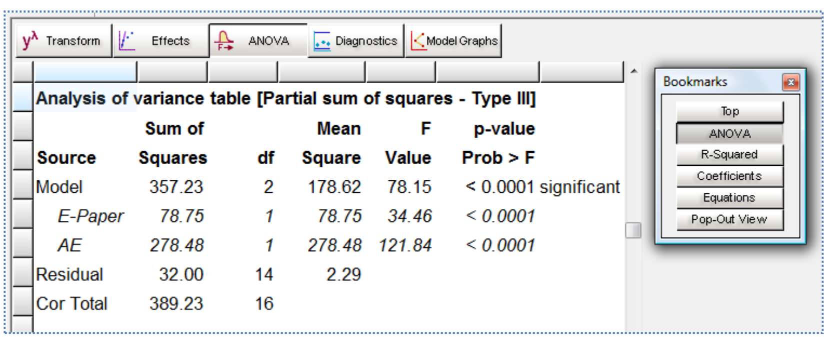
Now, on **Effects Tool** press the **Half-Normal** view and rope off the two outstandingly large effects as shown below.



Half-normal plot of subplot effects

Effects AE and E loom large, but be careful: Compare the bottom axis of this graph with the one done earlier for the whole-plot effects. Notice the range is several-fold less for the subplot effects. This reflects the comparatively high variance between repeated whole plot reactor setups (factors A through D) versus the variance within the subplot factor (changing paper type E).

Click the ***ANOVA*** button to see the significance of these subplot effects.



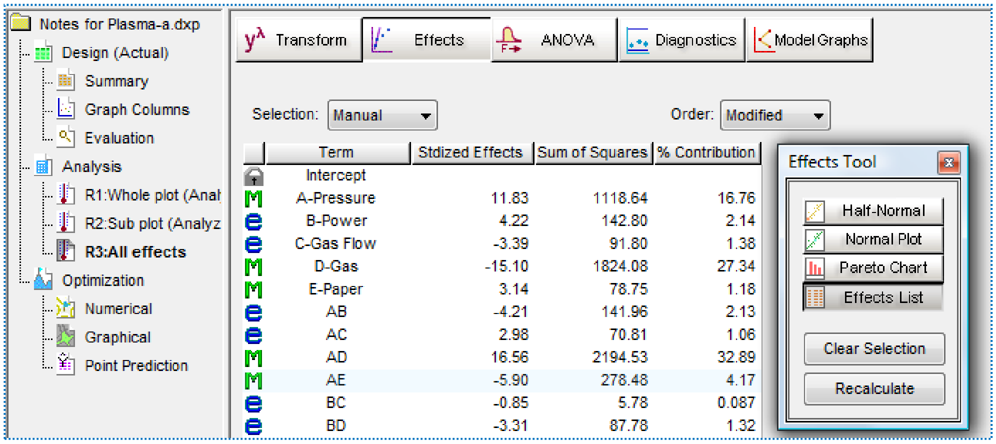
Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	357.23	2	178.62	78.15	< 0.0001	significant
E-Paper	*78.75*	*1*	*78.75*	*34.46*	*< 0.0001*	
AE	*278.48*	*1*	*278.48*	*121.84*	*< 0.0001*	
Residual	32.00	14	2.29			
Cor Total	389.23	16				

ANOVA for subplot model

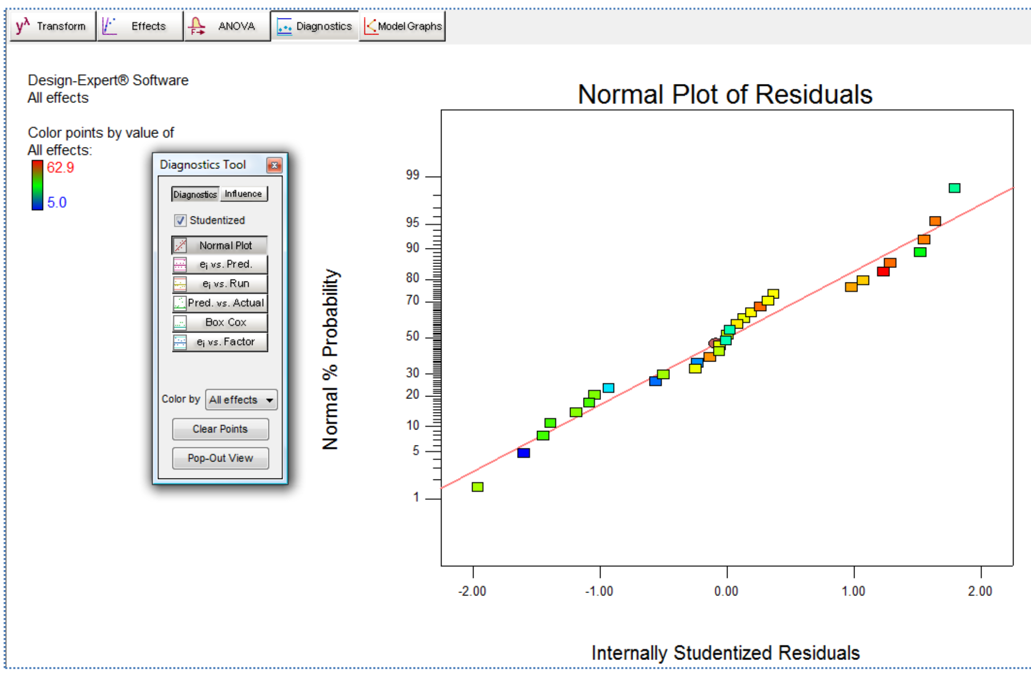
Do not look at the Diagnostics or the Model Graphs because the model is missing the whole plot effects. However, to preserve your ANOVA, select **File**, **Save** or simply click the save icon 🖫.

To diagnose the entire model and graph all the effects, click the **All effects** node for analysis. Then press the ***Effects*** button. On the floating tool, click **Effects List**. Double click terms **A**, **D**, **E**, **AD** and **AE** to model them (designation **M**). Terms A, D, and AD were found significant in the analysis of the whole plot. AE and E come from the analysis on the sub-plot. (Alternatively, you can right-click one or more highlighted terms and select them for your Model.)



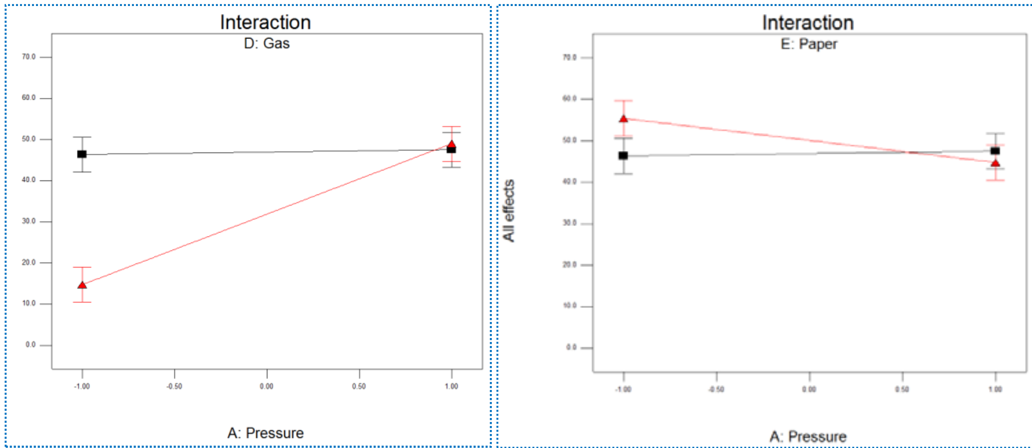
Final model selection for diagnostics and effect graphs

Skip through the ANOVA – all variances have already been analyzed via the breakdown by whole- and sub-plot terms. Press ahead to ***Diagnostics***.



Diagnostics for final model with all effects

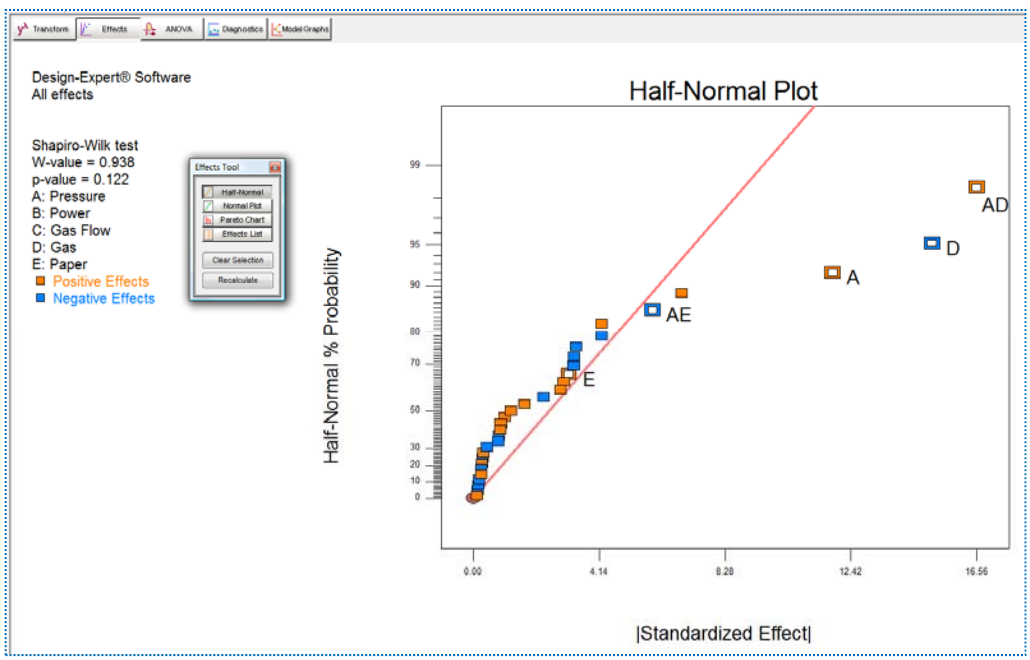
The model is now complete so it's OK to use these validation tools. Nothing looks amiss here, for example the normal plot of residuals line up nicely, so continue on to **Model Graphs**. Notice how much more the AD interaction affects the response versus the AE interaction.



Interaction AD (whole plot) on the left vs. interaction AE (subplot) at the right

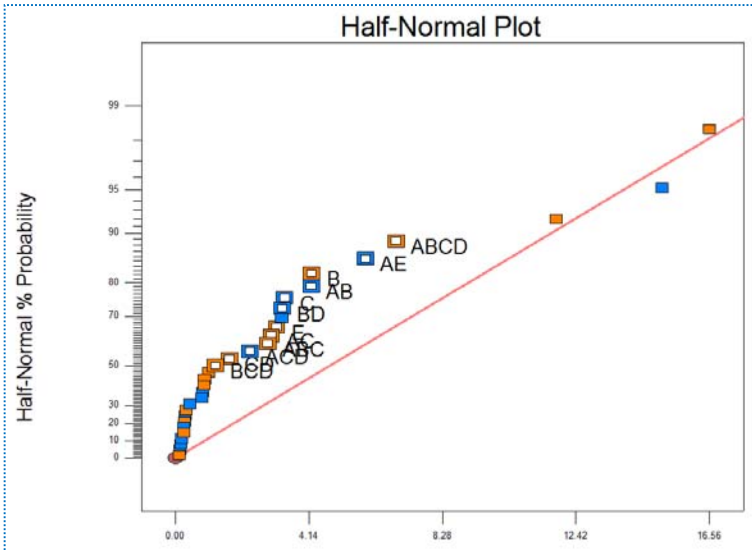
To preserve the modeling used for producing the model graphs, select **File**, **Save** or click the save icon 🖫.

The big question is: Would the significant effect of AE be obscured if the experimenters didn't recognize the split plot structure of their design? To determine the answer, go back to the ***Effects*** stage of the analysis flow and from the floating tool select the view **Half-Normal**.



Uncovering the AE interaction

If you click on the points near zero (to the left – the origin of the half-normal plot), you will identify mostly subplot effects (interactions involving factor E), which exhibit less variance (steeper slope). As illustrated below, then comes a grouping of effects not involving factor E – the whole plot terms, within which are found the effects of E and AE. These subplot effects are obscured by the variance between reactor setups (the whole plot)! This reflects the dual error structure of the split-plot design. The AE and E effects are buried in the error terms of the whole plot (factors A through D).



Effects AE and E buried in whole-plot error terms

Due to the split-plot design structure, the near-zero effects that normally line up on the half-normal plot become segregated into two groups – whole versus subplot, which makes it difficult to interpret without separating them as was done in this tutorial.

The manipulation of effect plots in this split-plot case uncovered a small but significant interaction that otherwise would've been obscured. In all likelihood, overlooking this effect makes no difference from a practical perspective because the big breakthrough effects (AD, A, D) are revealed.