Split-Plot General Factorial Tutorial

Introduction

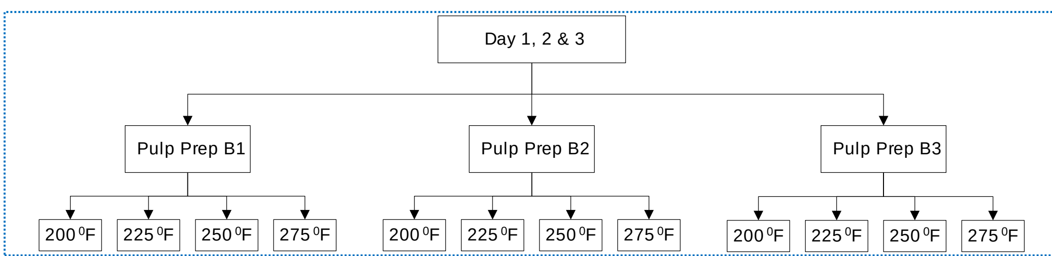
In some randomized block designs you must restrict the randomization. Otherwise it wouldn't be practical to perform the experiments. For example, temperature cannot be easily changed in many applications. The solution may be a "split-plot" design, which originated in the field of agriculture. Experimenters divided large areas of land, called "whole plots," into smaller "subplots" that could be treated separately. For example, they planted several crop varieties —one each per whole plot, and then applied different fertilizers on each subplot.

The analysis of a split-plot design is tricky, even for statisticians. It can be done with Design-Expert® software by properly designating effects in specific ways for subsequent analysis of variance. Proceed if you dare!

To illustrate how Design-Expert can be manipulated to do a split plot, let's follow an example from Montgomery's *Design and Analysis of Experiments*. A manufacturer wants to determine the effect of three methods of pulp preparation and four levels of cooking temperature on the tensile strength of paper. These twelve combinations can be conducted in one day.

To increase statistical power, the manufacturer decides to do three replicates of this twelve-run general factorial design over three days, which produces a total of 36 runs. However, it is not feasible to perform the experiment in a random fashion. Instead, on each of the three days, the experimenter produces a batch of pulp and divides it into four samples to be cooked at four different temperatures. This process is repeated for pulp prepared by the two other methods.

A flowchart of the experiment is shown below.



Flowchart of split plot experiment on papermaking process

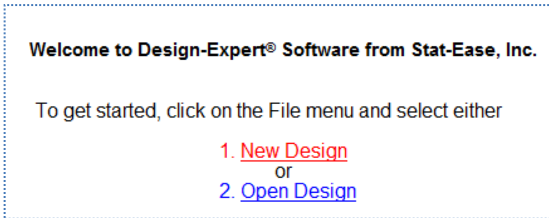
In statistical terms, the split plot experiment can be structured as:

- Whole plots for the three batches of pulp
- Sub-plots for the four samples cooked at four different temperatures.

You will set up this design as a general factorial. It will then need to be modified somewhat so it comes out in the correct order. Then you will perform an analysis, which gets quite tricky due to randomization restrictions.

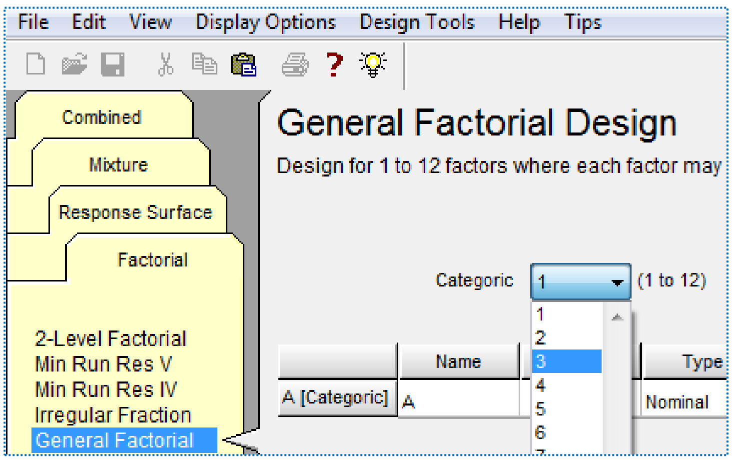
Design the Experiment

Let's build the design. This is done via the usual procedure for general factorials, but with days explicitly accounted for as a factor. From the Design-Expert welcome screen, click **New Design** (or click the blank-sheet icon 🗋 on your toolbar, or choose File, New Design).



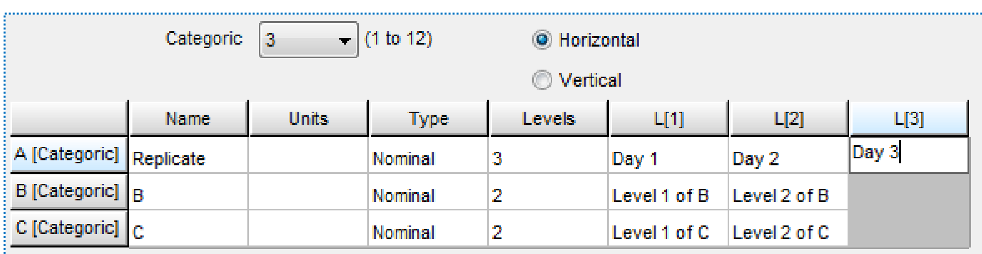
Opening a new design

Then from the default **Factorial** tab, click **General Factorial**. Choose **3** as the number of factors, as shown below.



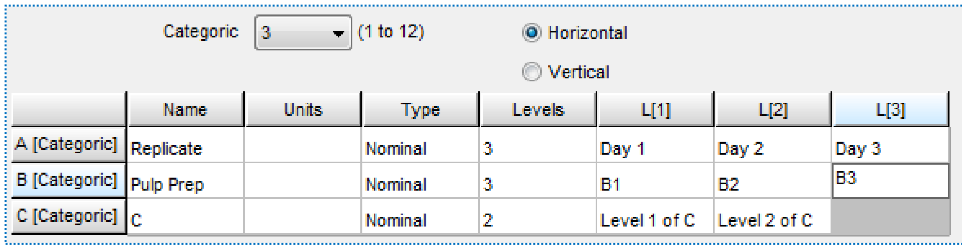
Setting up the general factorial design

Enter **Replicate** as your Factor A name. **Tab** to the **Levels** column and type **3** for the number of levels. **Tab** again and enter **Day 1**, **Day 2**, and **Day 3** as your **L(1)**, **L(2)**, and **L(3)** treatment names. Your screen should look like that below. (If your screen is oriented vertically, click on the Horizontal radio button to match the layout shown below).



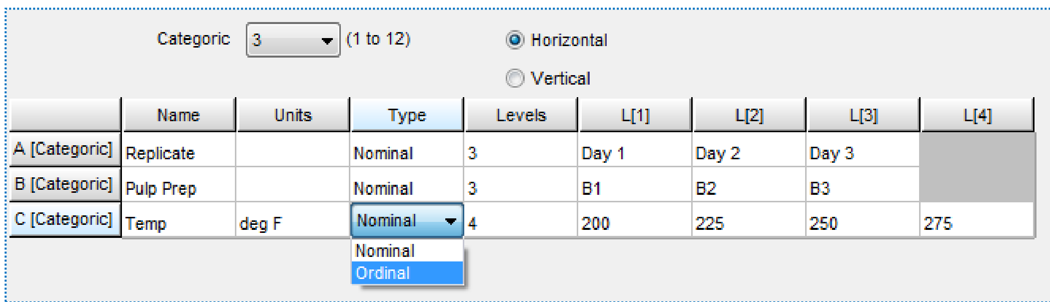
Entering first factor – days

Carry on with this screen by entering **Pulp Prep** as your Factor B name, and **3** for the number of levels. Change the treatment names to **B1**, **B2**, and **B3** as shown below.



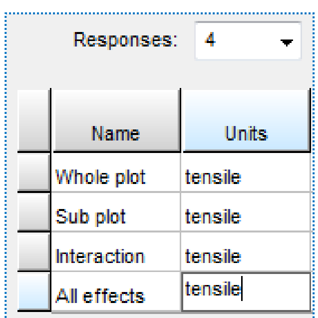
Second factor: Pulp prep

Still on the same screen, name your final (third) factor **Temp**, with **deg F** for units, **4** for the number of levels, and **200**, **225**, **250**, and **275** as your treatments. Change **Type** to **Ordinal**, thus denoting its numeric nature as shown below.



Last factor changed to "ordinal"

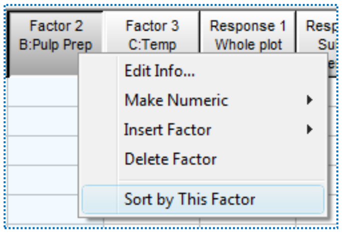
Press **Continue**, leave "1" for replicates as the default to produce 36 runs (36=3x3x4 levels from the three factors), and again press **Continue**. Although only one response has been measured in this experiment, it will be broken down four ways (to be explained later), so choose **4** responses and <u>enter the names shown below</u>. If you are adept at copying and pasting via Edit or shortcut keys, Ctrl-C and Ctrl-V, respectively, use these tools to save keystrokes when entering units as **tensile** as shown below.



Entering response name and units

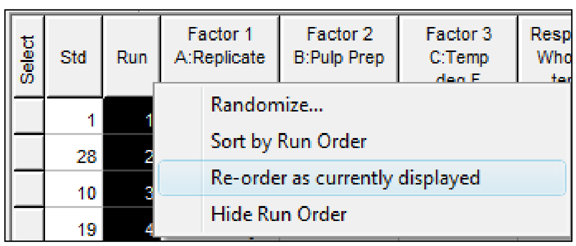
Don't bother completing any fields relating to calculating power for the design – simply press **Continue** to complete the design-building wizard.

Design-Expert now displays your design in completely random order. But obviously this cannot be performed as listed: Day 1 must precede day 2, all temperatures must run on one method of pulp preparation before starting the next, and so forth. To modify the order, first right click the **Pulp Prep** (Factor 2) column heading and choose **Sort by This Factor**.



Sorting by Pulp Prep method

Next, right click the **Replicate** (Factor 1) column and select **Sort by This Factor**. Now you see the subplot treatment settings (C - cooking temperature) randomized within the whole plot treatment (B - pulp preparation). To retain this order, right click on the **Run** column header and select Re-order as currently displayed.

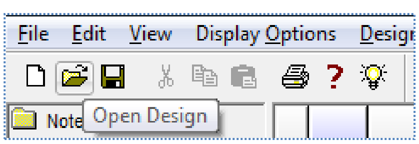


After sorting by Replicate select Run re-order as currently displayed

At this stage the design would normally be printed as a run-sheet for the actual experiment, with resulting responses of tensile strength recorded.

Analyze the Results

To avoid the trouble of typing in data, simply read in the responses via the 🗁 icon or via File, Open Design from the main menu.



Opening the design file to get the response data

Select the file named **Paper.dxp**. If this file does not appear, locate the data folder by finding the Design-Expert 8 data folder in the installed Stat-Ease program group under your All Programs menu. You can also download the tutorial files by going to the Help Menu – Tutorials. You should now see the data depicted in the following table, but with the tensile strength results duplicated four times in preparation for the various ways that it will be analyzed. Note that the whole plot factor “Pulp Prep” is randomized within days. This was done manually by randomizing the order of the numbers 1, 2 and 3 and then editing the run order within each day.

Run	A: Rep	B: Pulp Prep	C: Temp deg F	Tensile Strength
1	Day 1	B2	275	42
2	Day 1	B2	250	38
3	Day 1	B2	200	34
4	Day 1	B2	225	41
5	Day 1	B1	275	36
6	Day 1	B1	250	37
7	Day 1	B1	200	30
8	Day 1	B1	225	35
9	Day 1	B3	225	26
10	Day 1	B3	275	36
11	Day 1	B3	250	33
12	Day 1	B3	200	29
13	Day 2	B1	225	32
14	Day 2	B1	250	40
15	Day 2	B1	200	28
16	Day 2	B1	275	41
17	Day 2	B2	275	40
18	Day 2	B2	225	36
19	Day 2	B2	250	42
20	Day 2	B2	200	31
21	Day 2	B3	275	40
22	Day 2	B3	225	30
23	Day 2	B3	250	32
24	Day 2	B3	200	31
25	Day 3	B3	275	45
26	Day 3	B3	250	39
27	Day 3	B3	225	34
28	Day 3	B3	200	32
29	Day 3	B2	275	44
30	Day 3	B2	225	40
31	Day 3	B2	200	35
32	Day 3	B2	250	39
33	Day 3	B1	250	41
34	Day 3	B1	225	37
35	Day 3	B1	275	40
36	Day 3	B1	200	31

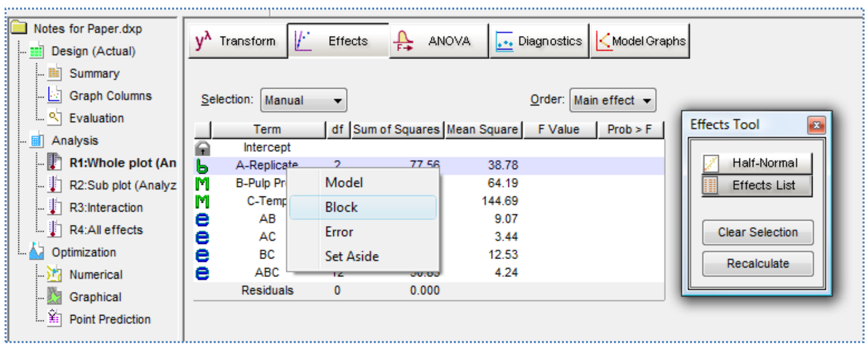
Response data for tensile strength of paper

The analysis of a split plot design in Design-Expert must be done somewhat manually. You need to create separate ANOVA's for the whole plot treatment (pulp prep), the subplot treatment (temperature) and the interaction between whole plot and subplot individually to get the correct statistical tests for each. Then you fit the full model in order to get meaningful diagnostics and model graphs (but you ignore the full model ANOVA!). To keep these various analyses straight, the tensile strength results have been copied four times over.

To analyze the whole plot treatment (B - pulp preparation) click on the Analysis node labeled **Whole plot** which can be found in the tree structure along the left of the main window. Then click on the ***Effects*** button displayed in the progressive toolbar. Design-Expert software supports four alternative effect designations:

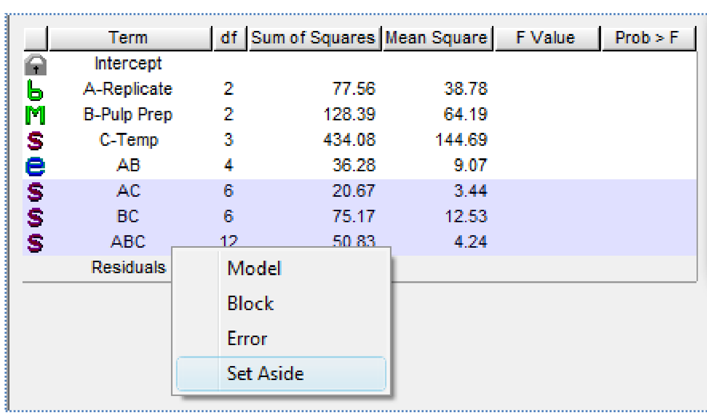
- Model ("M")
- Block ("b")
- Error ("e")
- Set Aside ("S")

The whole plot treatment (B) should be tested against the Replicate by Pulp Preparation interaction, i.e., the AB interaction. On the floating **Effects Tool** push the **Effects List** button. In order to include a block correction in the model, right click on factor **A** and choose **Block** (mark with "b").



Right-click menu for designating effects

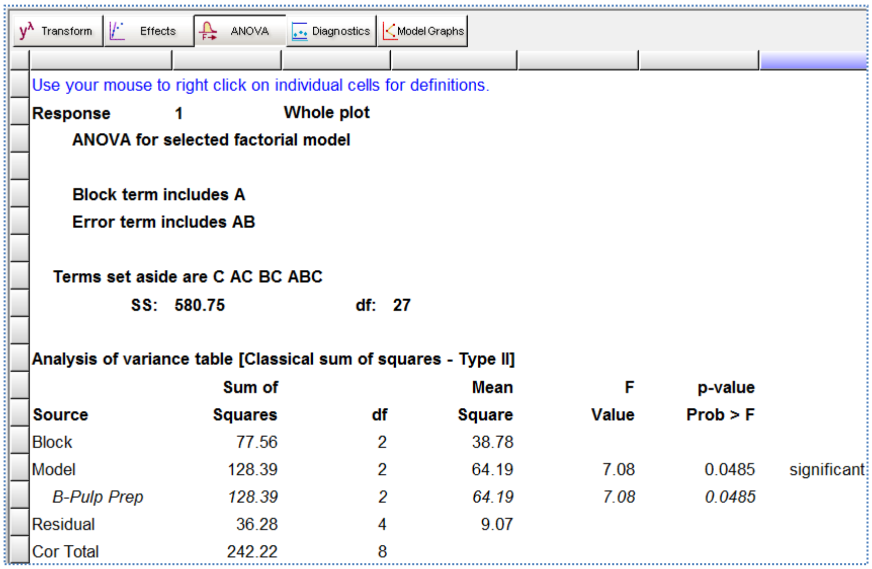
Right click on **C** and **Set Aside** (mark with "S"). Do the same for **AC**, **BC** and **ABC** (suggestion: try clicking on AC and shift-clicking on ABC to highlight all three effects, and then do one right-click to designate this block of effects to be ignored). Leave AB at its default designation as Error ("e"). Your screen should now match the illustration below.



Effects screen after designating effects for analyzing whole plot treatments

The designation of terms can be accomplished in numerous ways. Now is a good time to check out the options. You've seen the right-click menu approach to designating terms, which provides the broadest selection of options. You can either click on the one you want, or enter its first letter – "m," "b," "e" or "s." Try this! If you want to ignore a term, you can press the delete key. Go ahead and check this out. A double-click on the designation typically toggles a term back and forth from error (e) to model (M). Try this too. But before proceeding, return your selections to those shown in the figure above.

Click the ***ANOVA*** button. You can see that the whole plot treatment, pulp preparation (factor B), is significant. Ignore the remainder of the output for now, because this is only part of the picture.



Use your mouse to right click on individual cells for definitions.

Response 1 Whole plot

ANOVA for selected factorial model

Block term includes A

Error term includes AB

Terms set aside are C AC BC ABC

SS: 580.75 df: 27

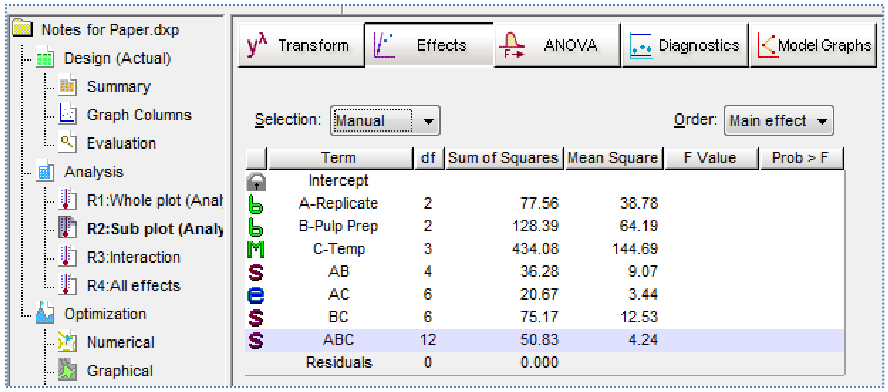
Analysis of variance table [Classical sum of squares - Type II]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Block	77.56	2	38.78			
Model	128.39	2	64.19	7.08	0.0485	significant
B-Pulp Prep	*128.39*	*2*	*64.19*	*7.08*	*0.0485*	
Residual	36.28	4	9.07			
Cor Total	242.22	8				

ANOVA for whole plot (main effect of B – the pulp preparation)

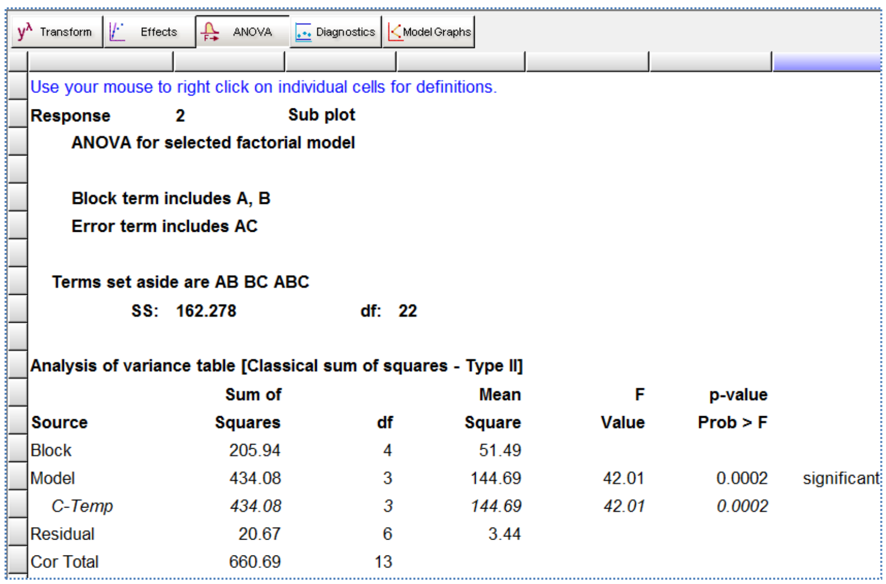
Do not look at the Diagnostics or the Model Graphs because the models are incomplete at this point. However, to preserve your ANOVA on the whole plot treatment (B – pulp preparation), select **File**, **Save As** and modify the name to **Paper-a.dxp** (or anything else you'd like that will leave the original tutorial file as-is).

To analyze the subplot treatment (C - temperature), click the **Sub plot** node for analysis, press the ***Effects*** button and the **Effects List** view. The subplot treatment (C) should be tested against the Replicate by Temperature interaction, i.e., the AC interaction. Drag over terms **A** and **B**, then right click and designate them **Block**. Leave C in the model ("M"). Right-click **AC** for the **Error**. All other terms (the interactions **AB**, **BC** and **ABC**) should be set to **Set Aside**. Your screen should now match the illustration shown below.



Designating effects for analyzing subplot treatments

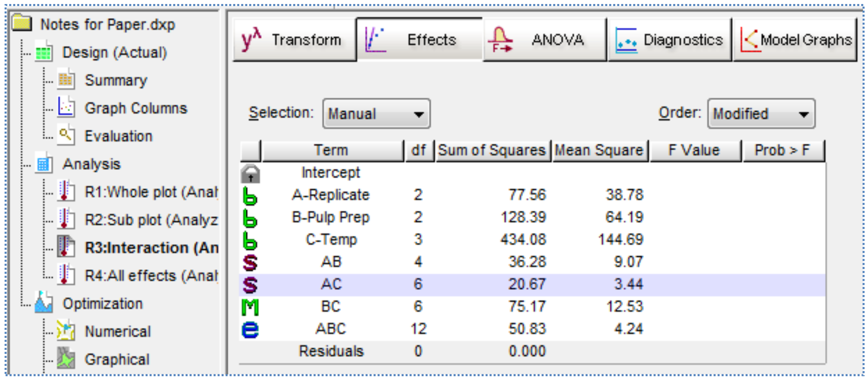
Click on the ***ANOVA*** button. You can see that the subplot treatment, cook temperature (factor C), is significant. Ignore the remainder of the output.



ANOVA for sub-plot

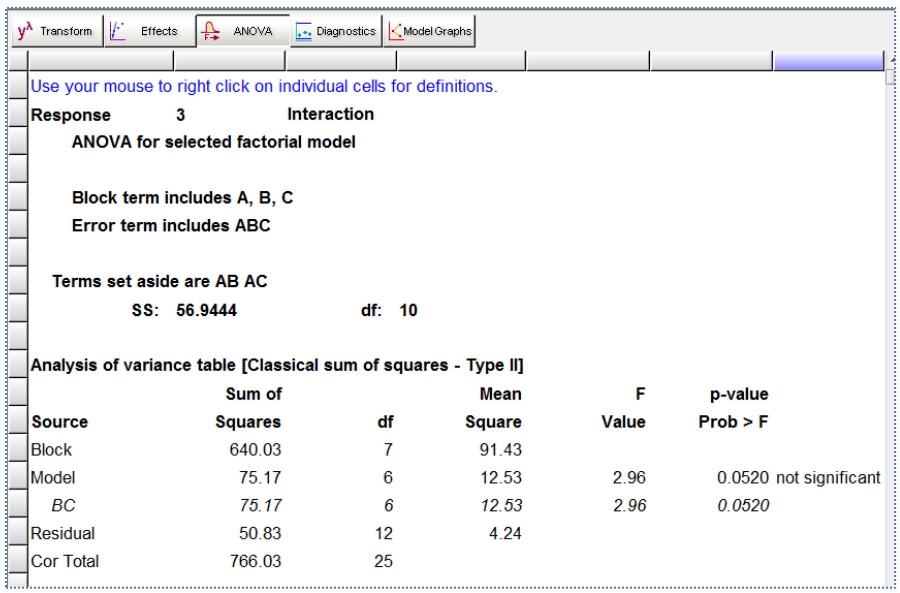
Do not look at the Diagnostics or the Model Graphs because the models are incomplete at this point. However, to preserve your ANOVA, select **File**, **Save** or simply click the save icon.

To analyze the whole plot by subplot treatment interaction, click on the **Interaction** node. Then press the ***Effects*** button and the **Effects List** view. Choose **BC** for the **Model**. This interaction should be tested against ABC, so leave the ABC term as error ("e"). But drag over factors **A**, **B** and **C** and right click them to **Block**. Lastly, set the **AB** and **AC** terms to **Set Aside**. Your screen should now match the following illustration.



Designating effects for whole plot by sub-plot interaction

Click on the ***ANOVA*** button.



ANOVA for whole plot by sub-plot interaction

The model in annotated view is labeled as "not significant." However, this is arbitrarily based on the Design-Expert program's default for a significance threshold of 0.05 (modifiable via Edit, Preferences, Math tab). Recall that earlier the effect of B (the whole plot treatment) was deemed significant at a p-value of 0.0485. The Prob > F for BC is only slightly higher at 0.052. Therefore, the experimenters decided to ignore the advice from the software, and consider BC to be significant.

Do not look at the Diagnostics or the Model Graphs because the models are incomplete at this point.

Before moving on to the final phase of the analysis, we want to provide some justification for the statistical analysis presented up to this point.

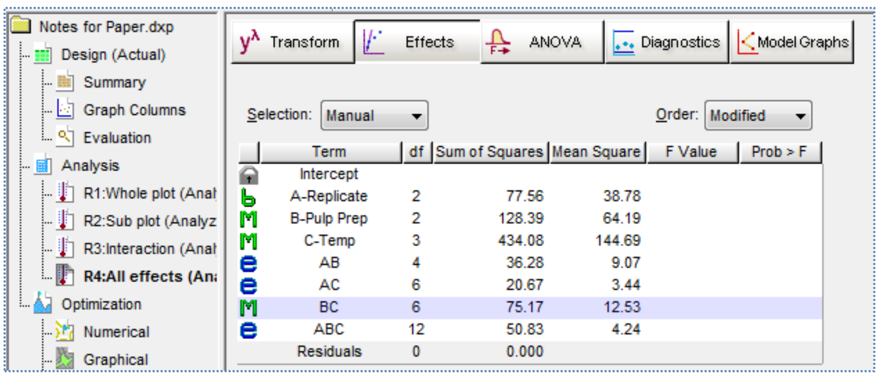
- In a blocked design the block-by-treatment interactions are used to estimate error.
- In a completely randomized blocked design all the block-by-treatment interactions are combined into a single estimate of error.

- Due to the restrictions on randomization in a split plot design, specific block-by-treatment interactions must be designated to estimate error for the particular treatment being tested. This is facilitated in this case by entering the blocks (day-by-day) as factor A.

Here's a recap of how you performed the analyses of variance (ANOVAs) for this experiment:

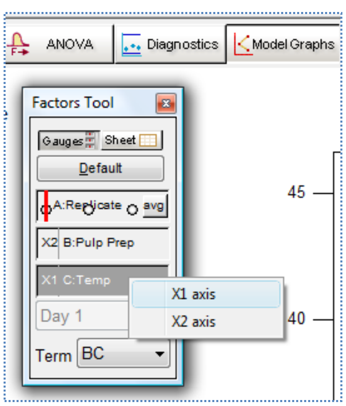
1. The whole plot treatment (B-pulp preparation) against the "block by pulp" (AB) interaction.
2. The sub-plot treatment (C-temperature) against the "block by temperature" (AC) interaction.
3. The pulp by temperature (BC) interaction against "block by pulp by temperature" (ABC) interaction.

To get meaningful diagnostics and model graphs, you now must fit the full model by clicking the **All effects** node. Click on ***Effects*** button and the **Effects List** view. Right-click **A** to **Block**. Leave B and C in the model. Also choose **BC** for the **Model**. Leave **AB**, **AC** and **ABC** at their defaults of **e** for error. Your screen should now match that shown below.



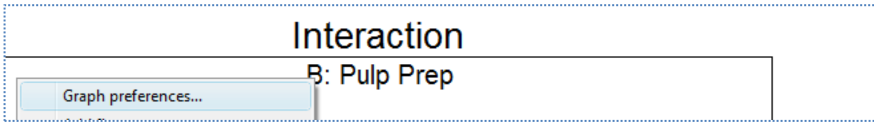
Fitting the full model for purposes of doing diagnostics and model graphs

Skip over the ANOVA (been there and done that) and click directly on the ***Diagnostics*** button. Examine all the graphs available via the Diagnostics tool. You should see nothing abnormal, so press ahead to ***Model Graphs***. On the **Factors Tool** right click over the **C: Temperature** bar and change it to the **X1 axis**.



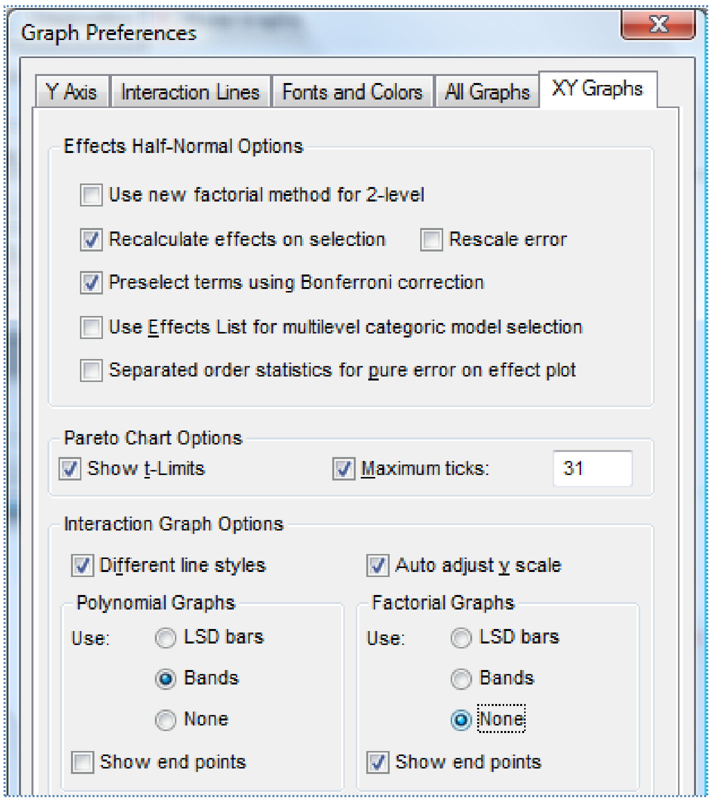
Changing axes

You now see an interaction plot of the continuous factor temperature at three discrete levels of pulp preparation. The LSD (least significant difference) bars that come up by default are incorrect, so they need to be turned off via right-clicking over the plot and selecting **Graph preferences**.



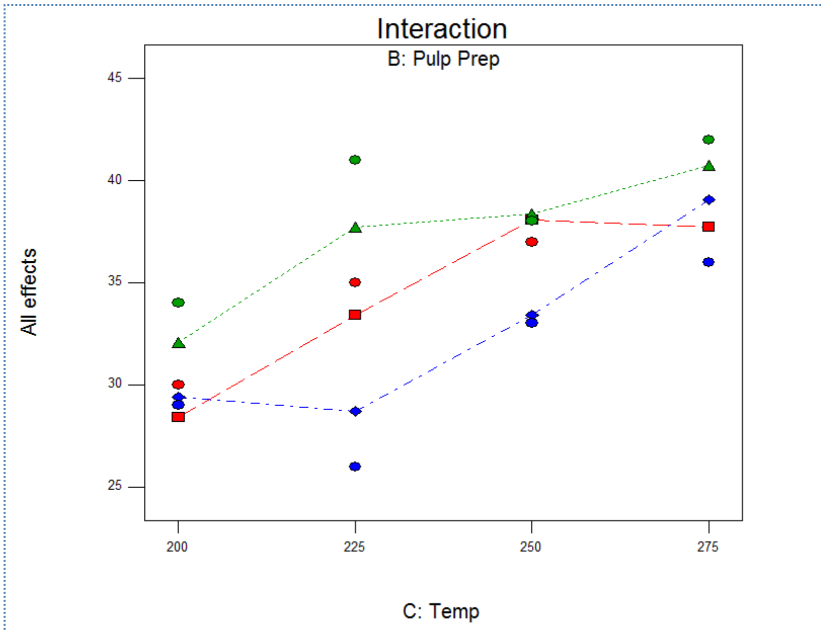
Effects graph – Graph preferences

Then click **XY Graphs** and under **Factorial Graphs** press the option for **None**.



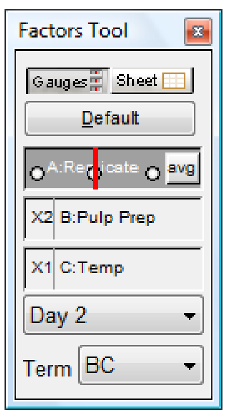
Turning off the LSD bars

OK this change (remember to reverse it back to LSD bars in the future when analyzing completely randomized designs). Now your interaction plot should look like that shown below.



Effects graph – interaction of temperature by pulp preparation on Day 1

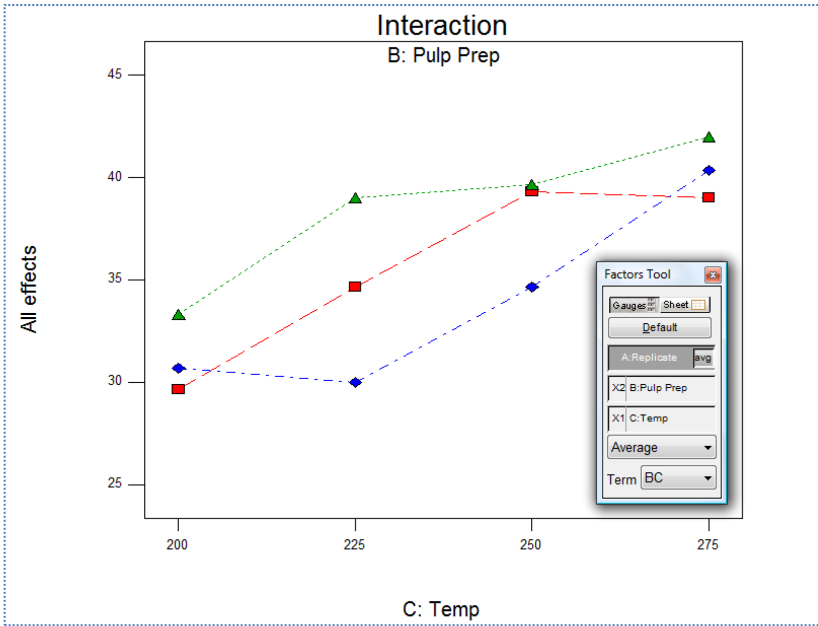
By default, the graph comes up for Day 1 (lowest level of factor A). On the **Factors Tool** press the second button on **Replicate** to see the predicted results for Day 2. (Note the red line through it in the figure below. Click on the circle in that position).



Setting Replicate to Day 2

Notice how the graph shifts very slightly. Click on any point to get the identification. The round symbols represent actual values, which are displayed to the left of the plot when you click the point. The other symbols (triangle, square and diamond in varying colors) show the predicted means. Click any of these points to get the value for tensile strength.

Now press the third button to see the predicted results for Day 3. However, since time cannot be controlled, it might be best to show the BC interaction at an average of the Days. As shown below, select **Average**.



Setting replicates to average level

From this graph we can see pulp preparation B2 (symbolized by the green triangles) will maximize tensile strength. Evidently it will be most robust to temperature changes, especially in the temperature range of 225 to 275 degrees F. This relationship holds true regardless of which day (replicate) you select (the buttons on the tool palette).

To preserve the modeling used to produce the plots, select **File**, **Save** or click the save icon.

This is the end of the story, but as a postscript, you may wonder if it was worth all the bother to properly account for the split-plot nature of this experiment. Click back to the ANOVA for this "All effects" analysis and then back to the ones for "Whole", "Sub plot" and "Interaction." The differences are relatively subtle. In other words, in this particular case, an experimenter who unwittingly ignored the restrictions in randomization caused by the split plot may have arrived at the same general conclusions shown here. In any case, it is always wise to assess the practical importance of the observed effects and, if warranted, follow up by doing confirmation runs to ensure that predictions from the model will be relatively accurate.